

Time series and error analysis

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Material from T. A. Herring, R. W. King, M. A. Floyd (MIT) and S. C. McClusky (now ANU)

http://web.mit.edu/mfloyd/www/courses/gg/201705_Bristol/

Issues in GPS Error Analysis

- What are the sources of the errors ?
- How much of the error can we remove by better modeling ?
- Do we have enough information to infer the uncertainties from the data ?
- What mathematical tools can we use to represent the errors and uncertainties ?

Determining the Uncertainties of GPS Parameter Estimates

- Rigorous estimate of uncertainties requires full knowledge of the error spectrum—both temporal and spatial correlations (never possible)
- Sufficient approximations are often available by examining time series (phase and/or position) and reweighting data
- Whatever the assumed error model and tools used to implement it, external validation is important

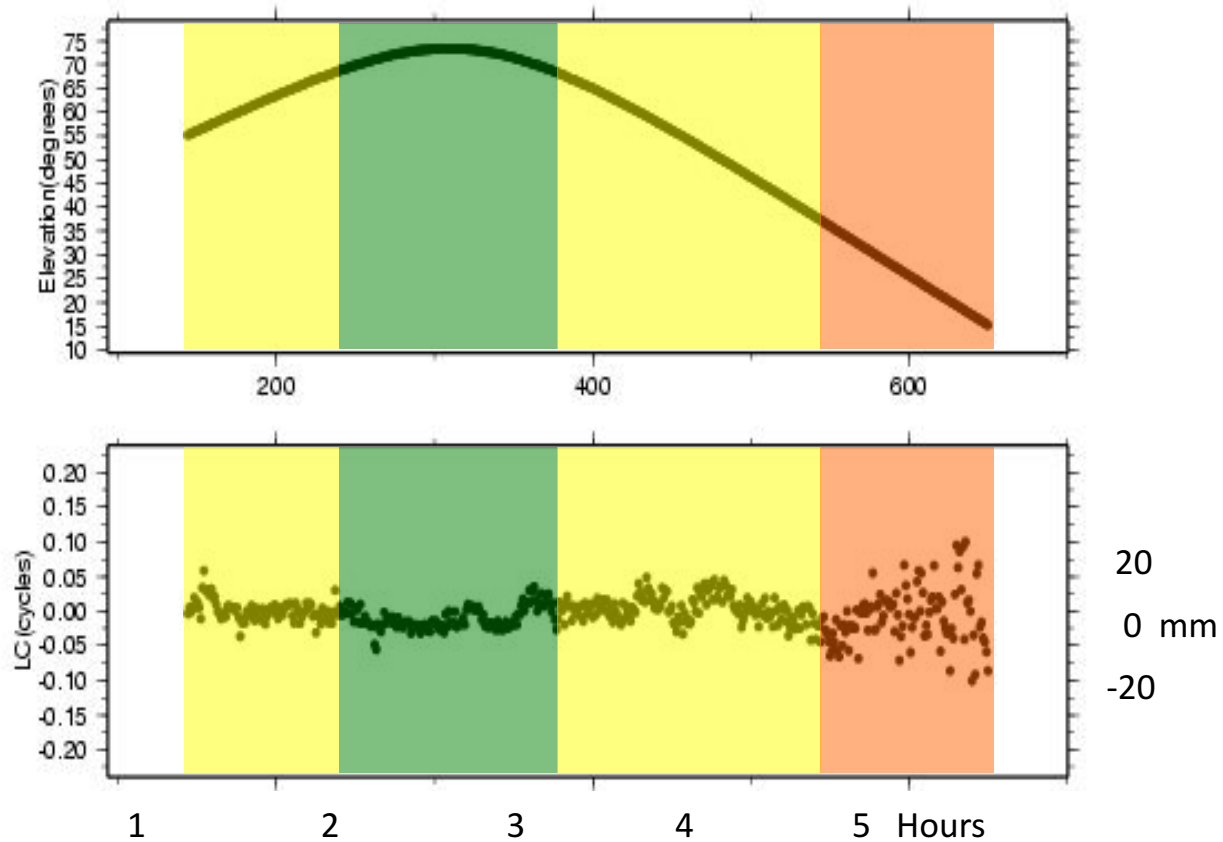
Tools for Error Analysis in GAMIT/GLOBK

- GAMIT: AUTCLN reweight = Y (default) uses phase rms from postfit edit to reweight data with constant + elevation-dependent terms
- GLOBK
 - rename (eq_file) _XPS or _XCL to remove outliers
 - sig_neu adds white noise by station and span; best way to “rescale” the random noise component; a large value can also substitute for _XPS/_XCL renames for removing outliers
 - mar_neu adds random-walk noise: principal method for controlling velocity uncertainties
 - In the gdl files, can rescale variances of an entire h-file: useful when combining solutions from with different sampling rates or from different programs (Bernese, GIPSY)
- Utilities
 - tsview and tsfit can generate _XPS commands graphically or automatically
 - grw and vrw can generate sig_neu commands with a few key strokes
 - FOGMEx (“realistic sigma”) algorithm implemented in tsview (MATLAB) and tsfit/ensum; sh_gen_stats generates mar_neu commands for globk based on the noise estimates
 - sh_plotvel (GMT) allows setting of confidence level of error ellipses
 - sh_tshist and sh_velhist (GMT) can be used to generate histograms of time series and velocities.

Sources of Error

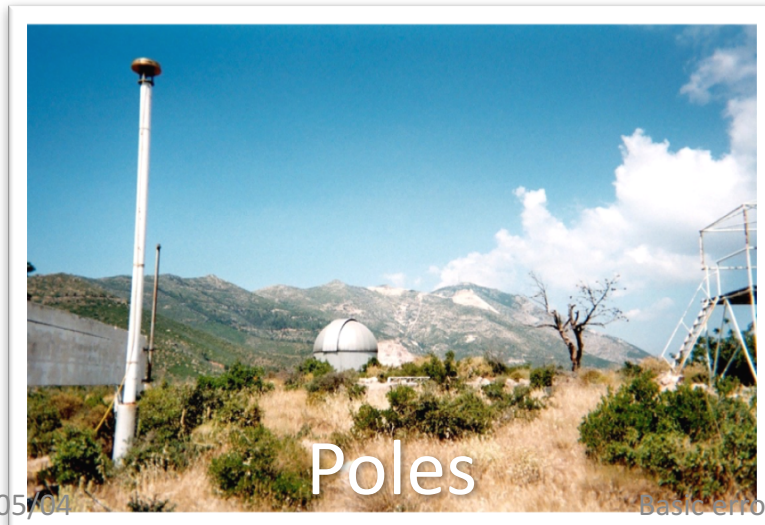
- Signal propagation effects
 - Receiver noise
 - Ionospheric effects
 - Signal scattering (antenna phase center / multipath)
 - Atmospheric delay (mainly water vapor)
- Unmodeled motions of the station
 - Monument instability
 - Loading of the crust by atmosphere, oceans, and surface water
- Unmodeled motions of the satellites

Characterizing Phase Noise



Elevation angle and phase residuals for single satellite

Fixed antennas



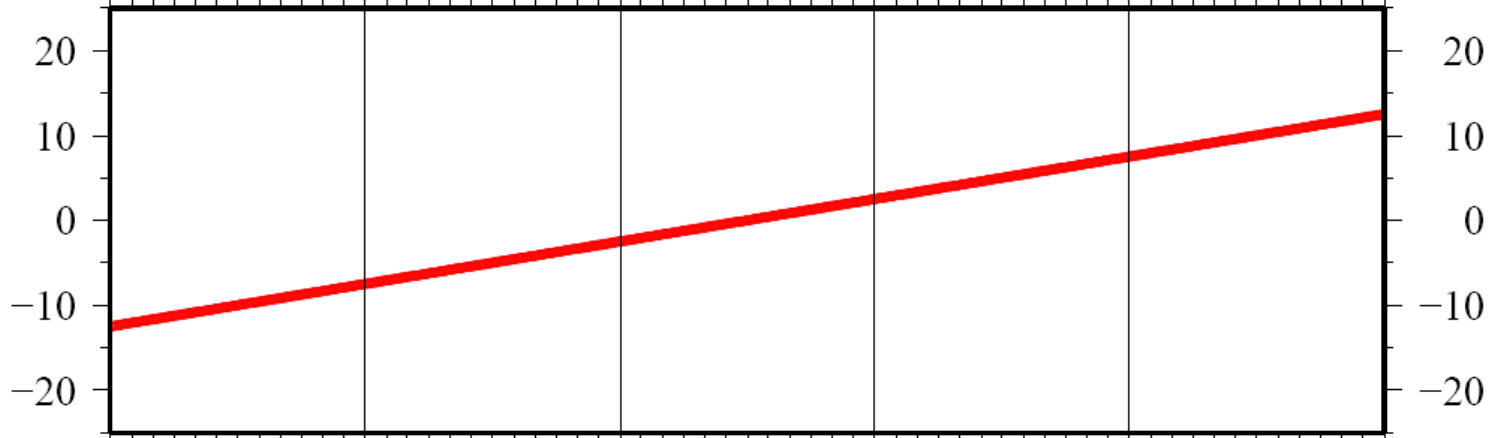
Time series characteristics

Time series components

observed position (linear) velocity term

$$x^i = x_0^i + v^i (t - t_0)$$

initial position

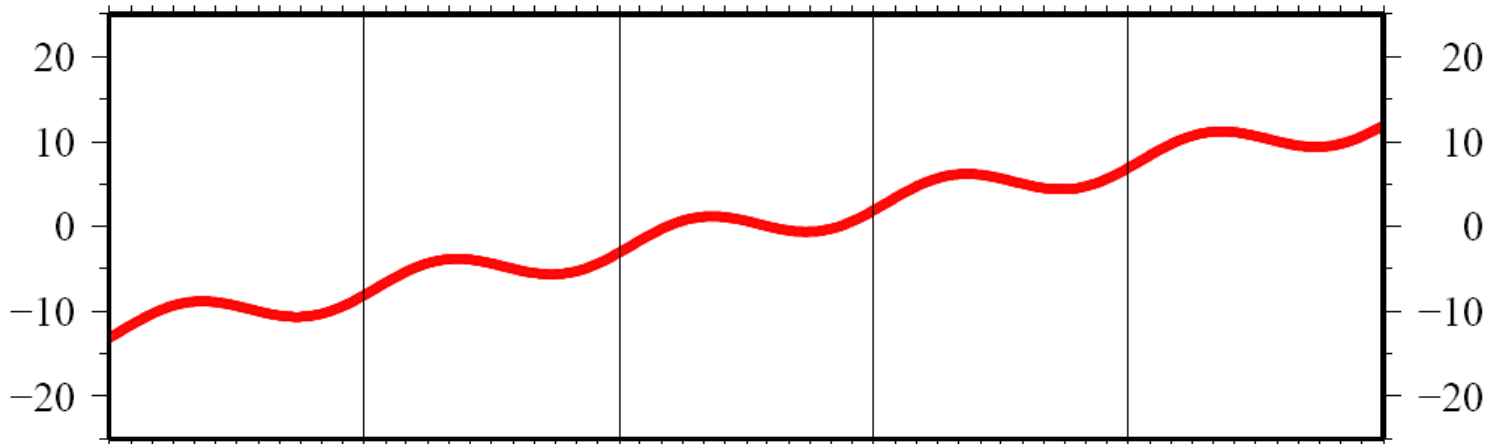


Time series components

observed position (linear) velocity term

$$x^i = x_0^i + v^i(t - t_0) + A_0^i \cos\left(\frac{2\pi(t - t_0)}{T_0} - \tau_0\right)$$

initial position annual period sinusoid



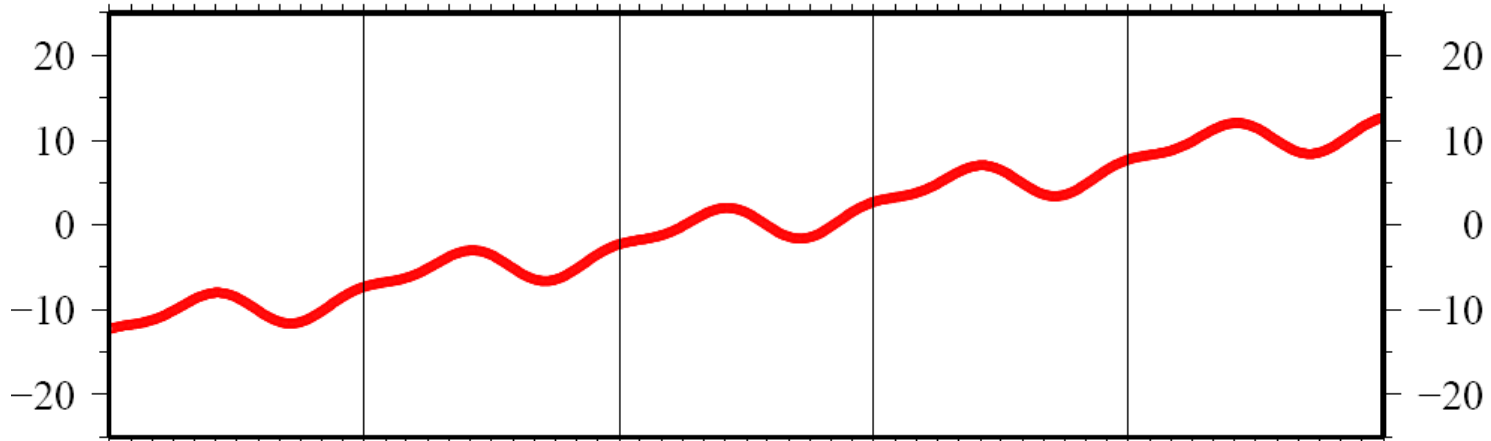
Time series components

observed position (linear) velocity term

$$x^i = x_0^i + v^i(t - t_0) + \underbrace{A_0^i \cos\left(\frac{2\pi(t - t_0)}{T_0} - \tau_0\right)}_{\text{annual period sinusoid}} + \underbrace{A_1^i \cos\left(\frac{2\pi(t - t_0)}{T_1} - \tau_1\right)}_{\text{semi-annual period sinusoid}}$$

initial position

seasonal term



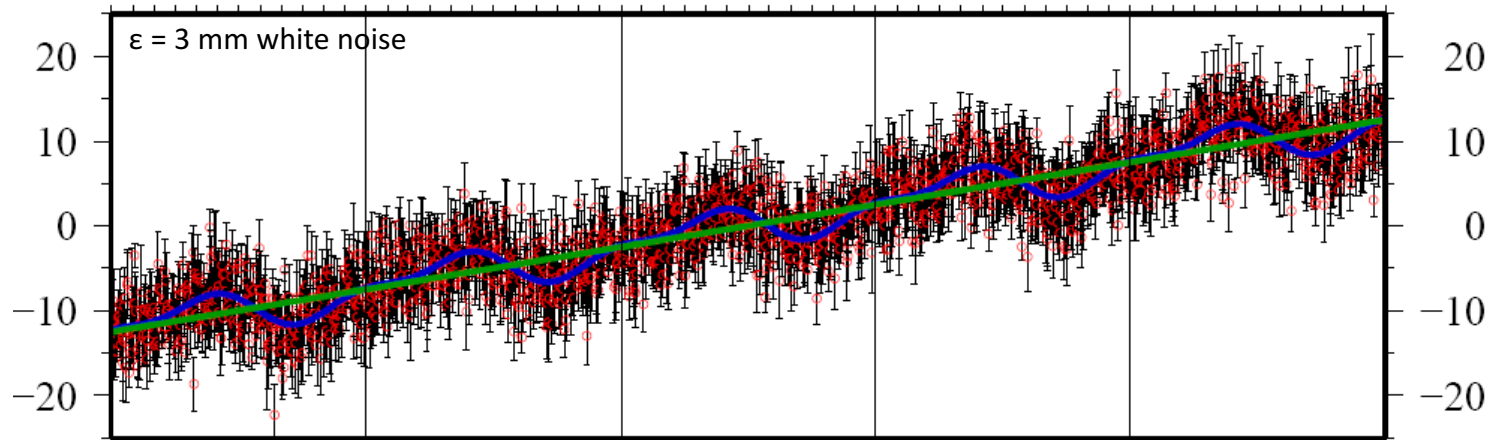
Time series components

observed position (linear) velocity term

$$x^i = x_0^i + v^i(t - t_0) + \underbrace{A_0^i \cos\left(\frac{2\pi(t - t_0)}{T_0} - \tau_0\right)}_{\text{annual period sinusoid}} + \underbrace{A_1^i \cos\left(\frac{2\pi(t - t_0)}{T_1} - \tau_1\right)}_{\text{semi-annual period sinusoid}} + \varepsilon$$

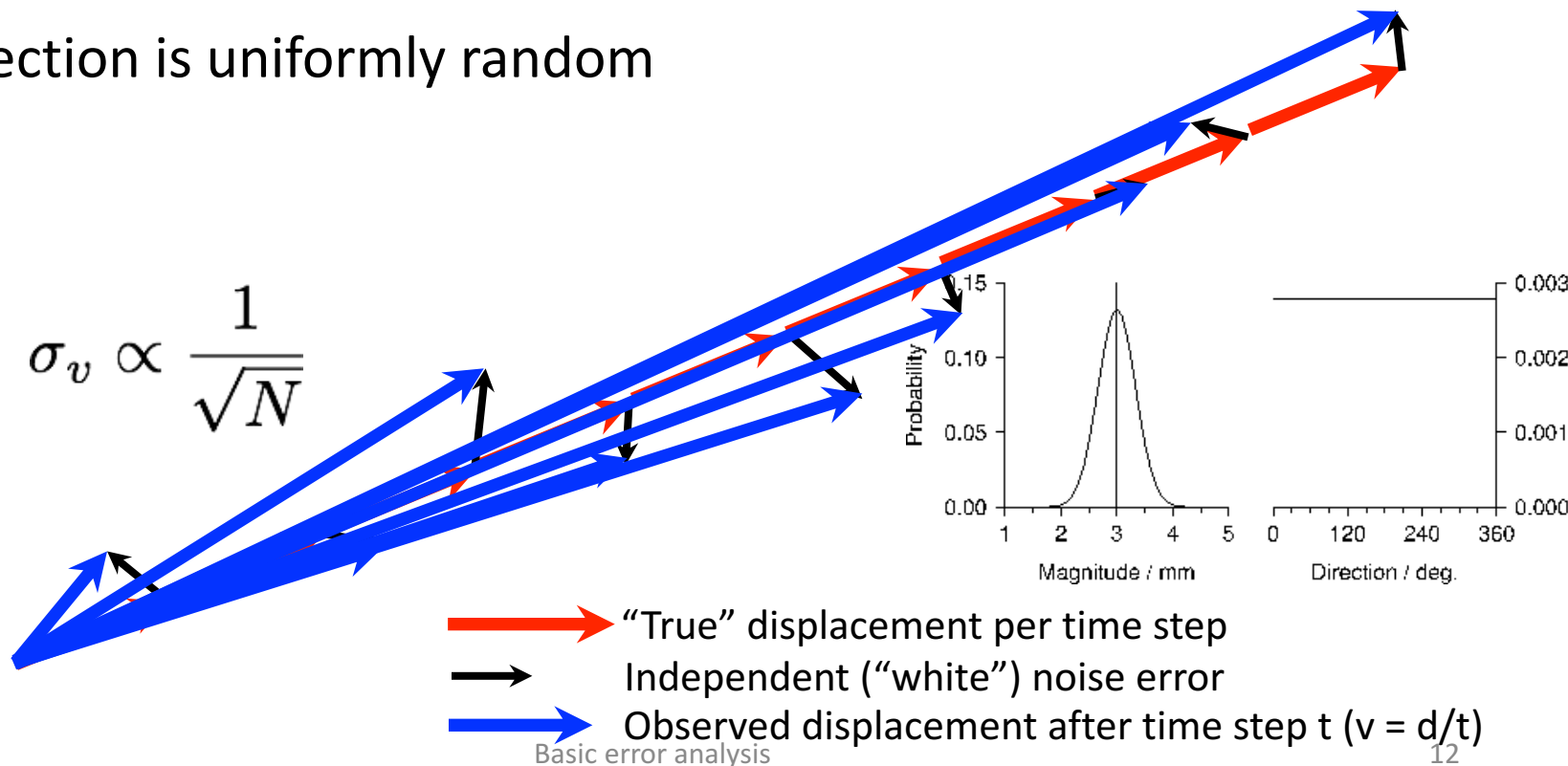
initial position

seasonal term



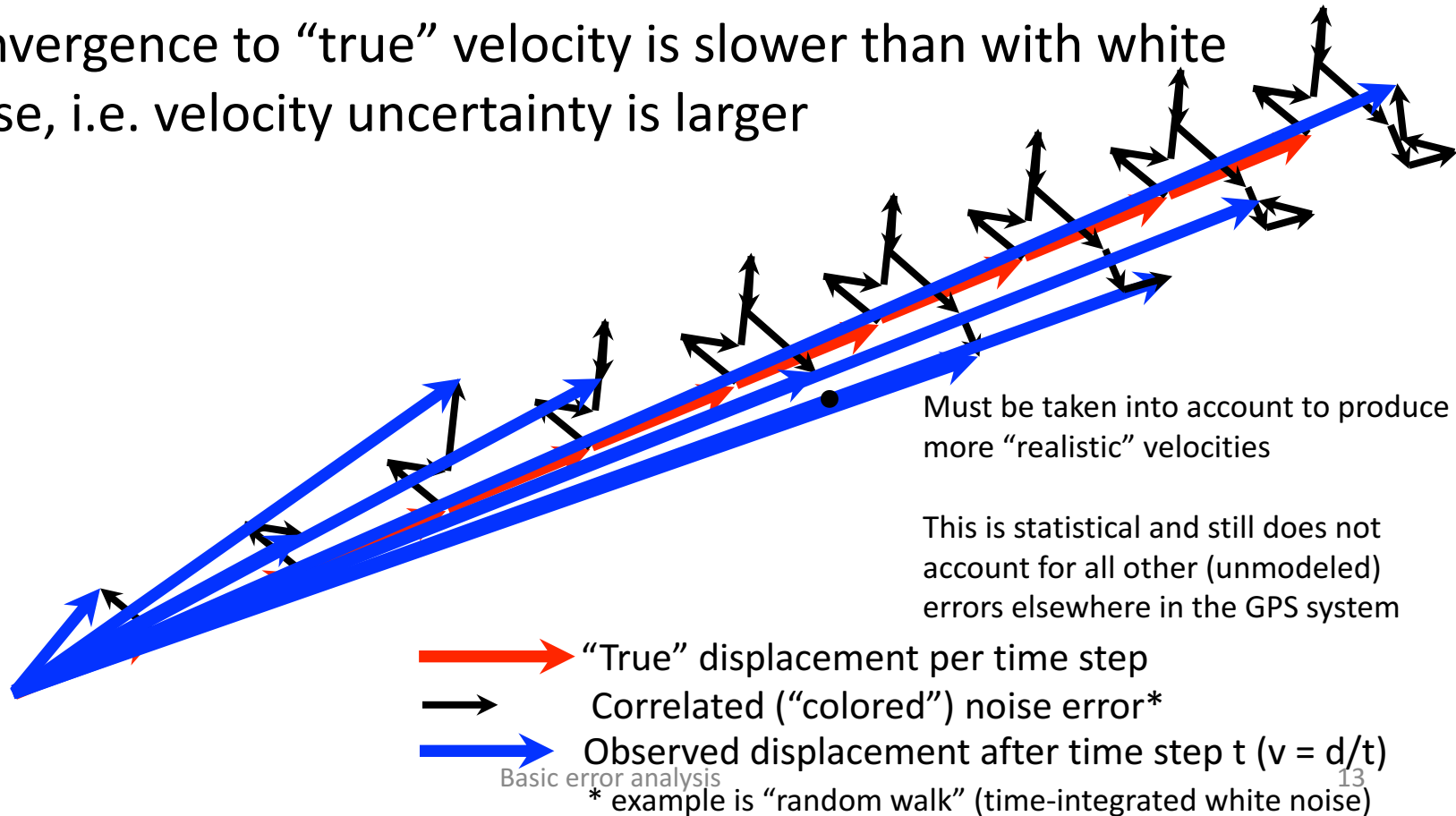
“White” noise

- Time-independent (uncorrelated)
- Magnitude has continuous probability function, e.g. Gaussian distribution
- Direction is uniformly random

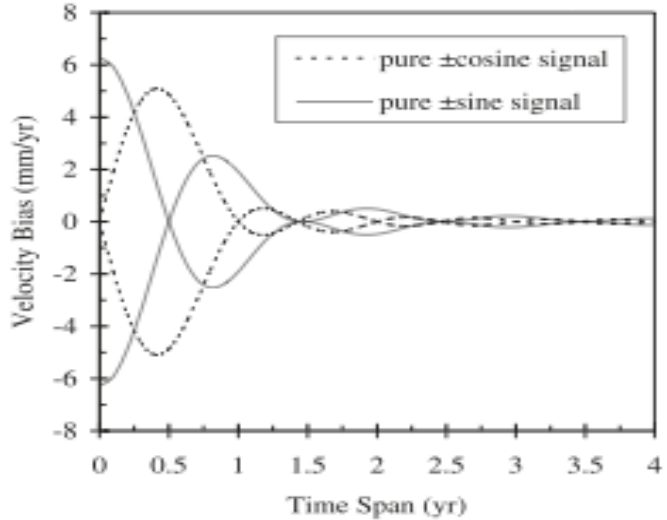


“Colored” noise

- Time-dependent (correlated): power-law, first-order Gauss-Markov, etc
- Convergence to “true” velocity is slower than with white noise, i.e. velocity uncertainty is larger

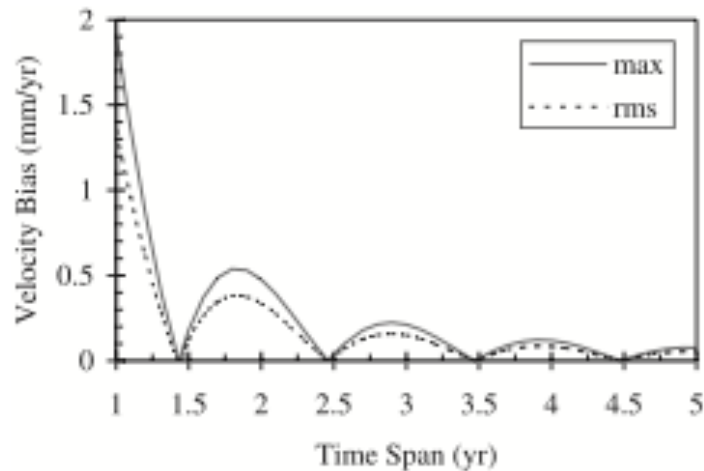


Velocity Errors due to Seasonal Signals in Continuous Time Series



Theoretical analysis of a continuous time series by *Blewitt and Lavallee [2002, 2003]*

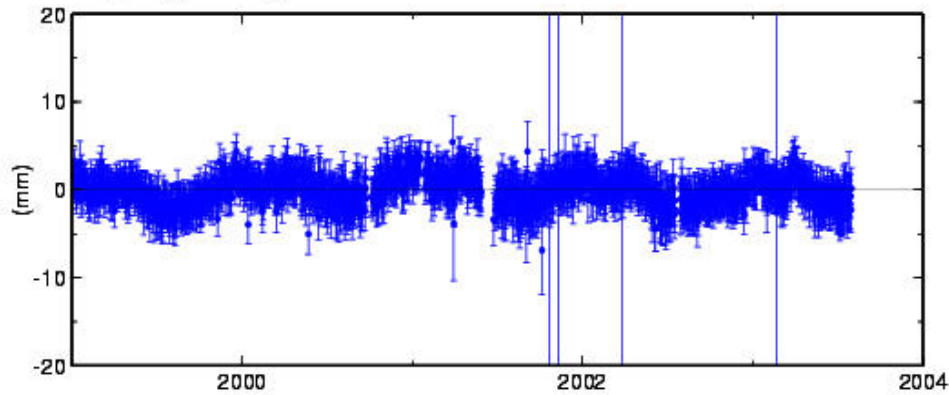
Top: Bias in velocity from a 1mm sinusoidal signal in-phase and with a 90-degree lag with respect to the start of the data span



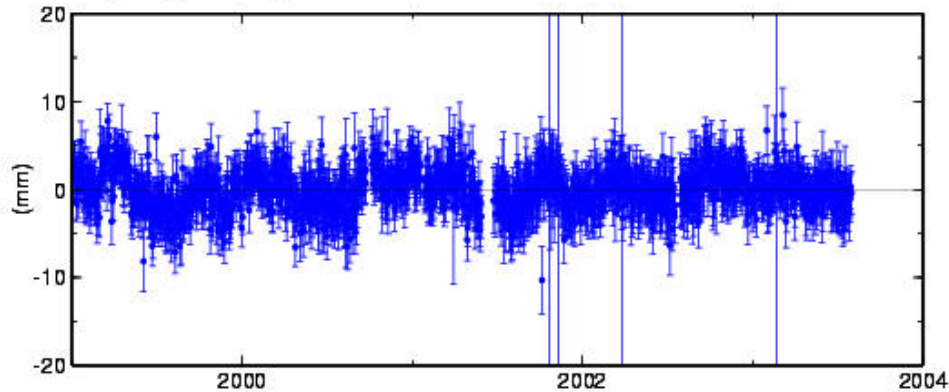
Bottom: Maximum and rms velocity bias over all phase angles

- The minimum bias is NOT obtained with continuous data spanning an even number of years
- The bias becomes small after 3.5 years of observation

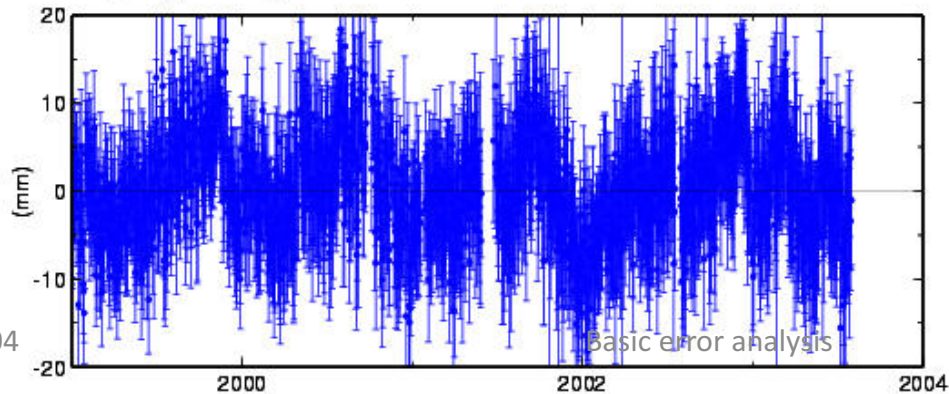
BURN North Offset 4762193.218 m
rate(mm/yr)= 1.39 ± 0.04 nrms= 0.69 wrms= 1.5 mm # 1578



BURN East Offset 19785454.795 m
rate(mm/yr)= -1.43 ± 0.05 nrms= 0.86 wrms= 2.1 mm # 1578

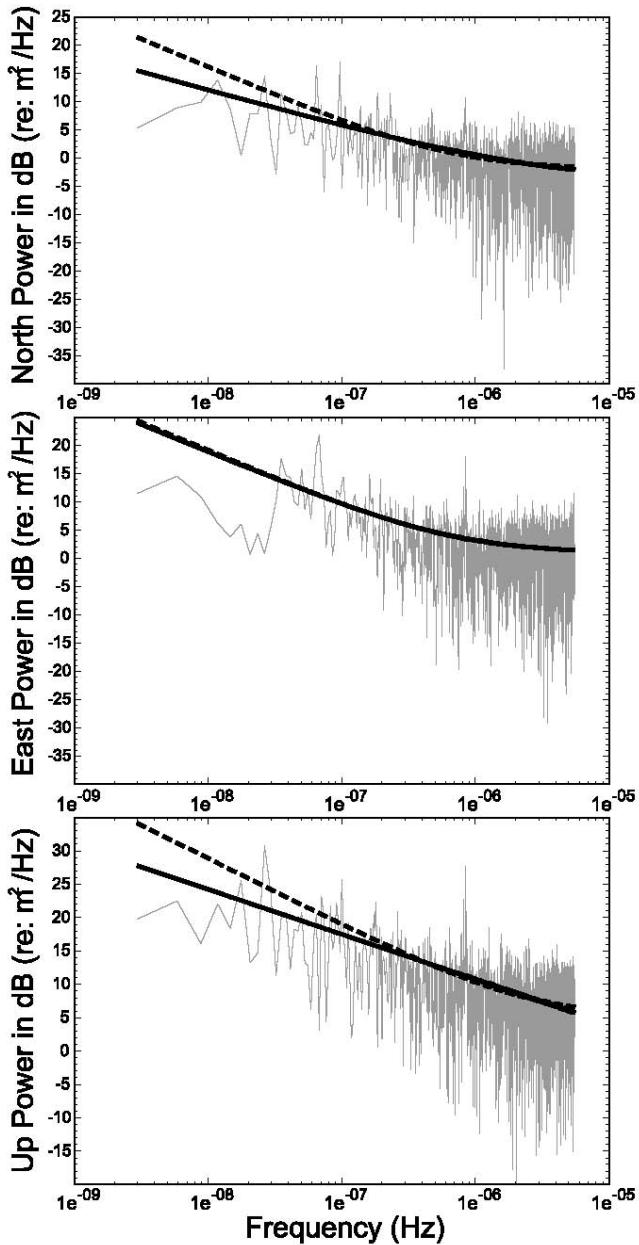


BURN Up Offset 1180.839 m
rate(mm/yr)= -1.62 ± 0.13 nrms= 0.79 wrms= 5.5 mm # 1578



Characterizing the Noise in Daily Position Estimates

Note temporal correlations of 30-100 days and seasonal terms



Spectral Analysis of the Time Series to Estimate an Error Model

Figure 5 from *Williams et al* [2004]: Power spectrum for common-mode error in the SOPAC regional SCIGN analysis. Lines are best-fit WN + FN models (solid=mean ampl; dashed=MLE)

Note lack of taper and misfit for periods > 1 yr

Summary of Spectral Analysis Approach

- Power law: slope of line fit to spectrum
 - 0 = white noise
 - -1 = flicker noise
 - -2 = random walk
- Non-integer spectral index (e.g. “fraction white noise” $\rightarrow 1 > k > -1$)
- Good discussion in Williams [2003]
- Problems:
 - Computationally intensive
 - No model captures reliably the lowest-frequency part of the spectrum

CATS (Williams, 2008)

- Create and Analyze Time Series
- Maximum likelihood estimator for chosen model
 - Initial position and velocity
 - Seasonal cycles (sum of periodic terms) [optional]
 - Exponent of power law noise model
- Requires some linear algebra libraries (BLAS and LAPACK) to be installed on computer (common nowadays, but check!)

Hector (Bos et al., 2013)

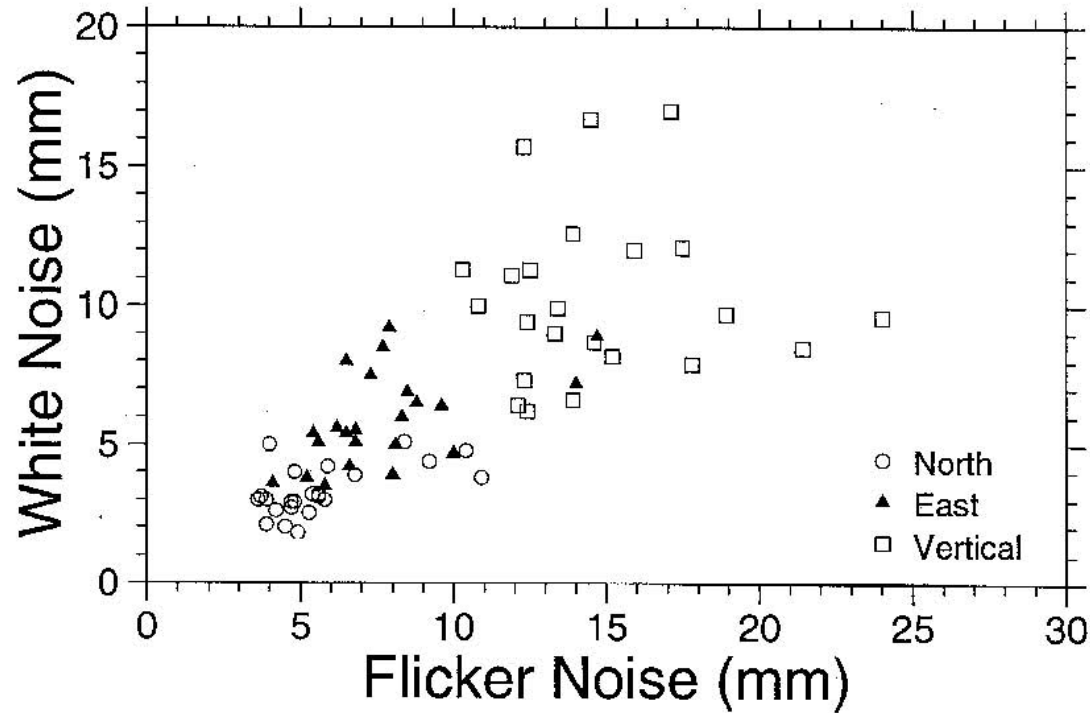
- Much the same as CATS but faster algorithm
- Maximum likelihood estimator for chosen model
 - Initial position and velocity
 - Seasonal cycles (sum of periodic terms) [optional]
 - Exponent of power law noise model
 - Also
- Requires ATLAS linear algebra libraries to be installed on computer
- Linux package available but tricky to install from source due to ATLAS requirement

sh_cats/sh_hector

- Scripts to aid batch processing of time series with CATS or Hector
- Requires CATS and/or Hector to be pre-installed
- Outputs
 - Velocities in “.vel”-file format
 - Equivalent random walk magnitudes in “mar_neu” commands for sourcing in globk command file
- Can take a *long* time!

Short-cut (Mao et al, 1998):

Use white noise statistics (wrms) to predict the flicker noise



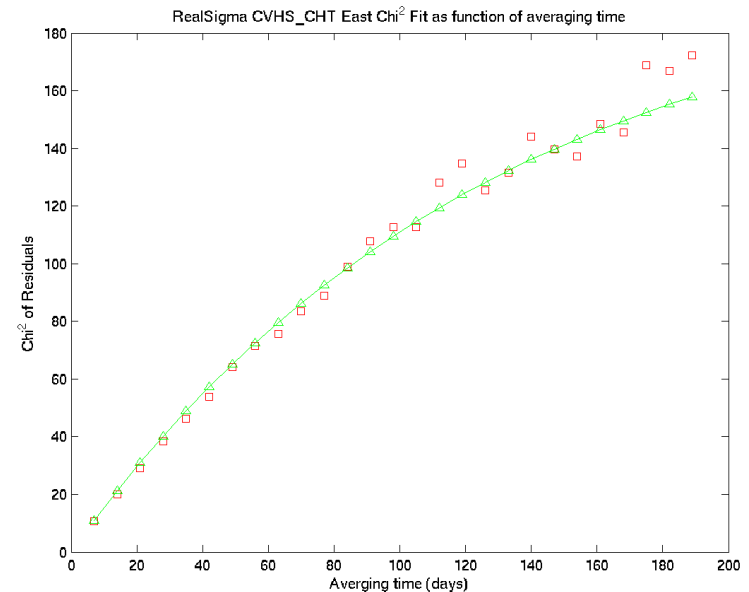
White noise vs flicker noise from *Mao et al.* [1999] spectral analysis of 23 global stations

“Realistic Sigma” Algorithm for Velocity Uncertainties

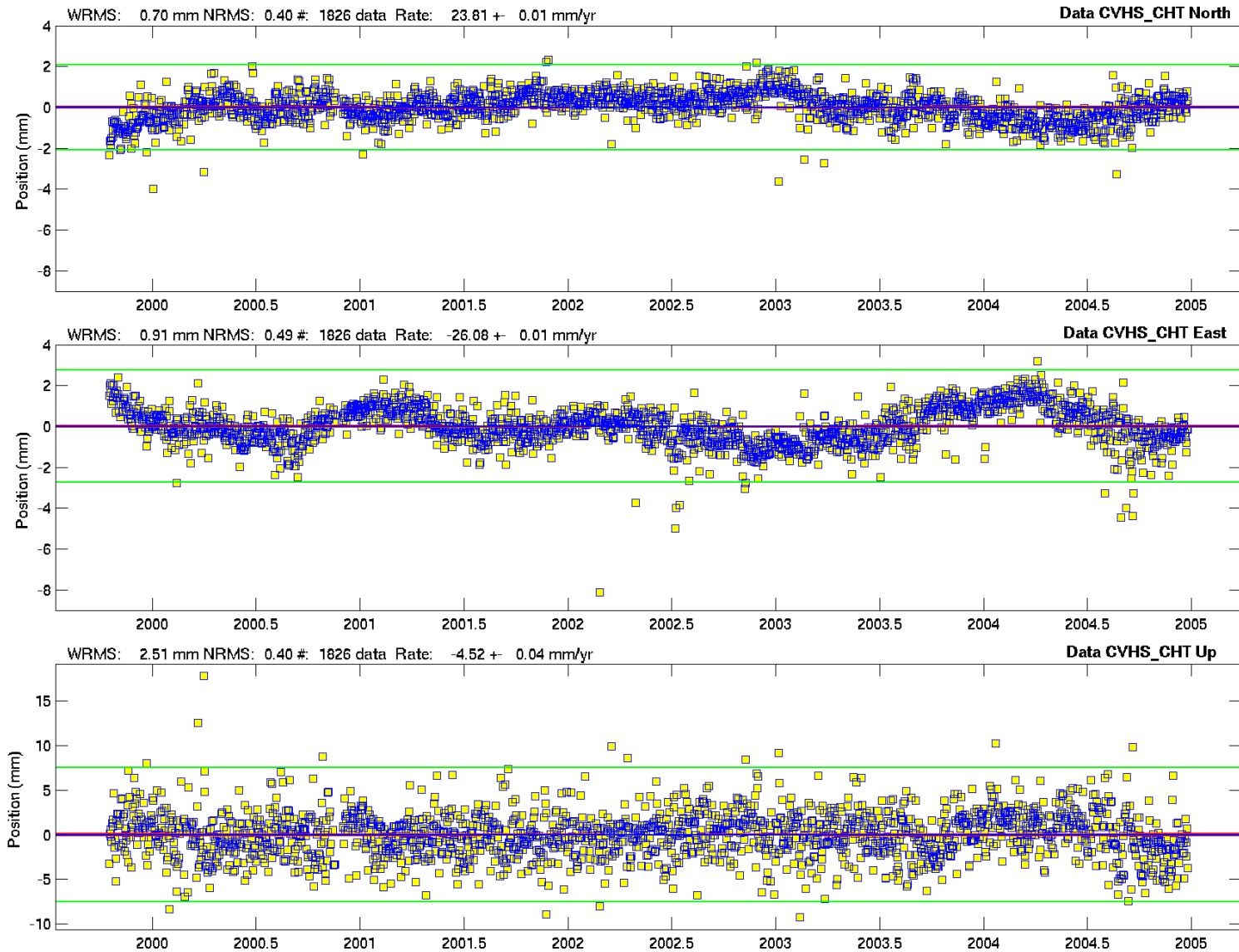
- Motivation: computational efficiency, handle time series with varying lengths and data gaps; obtain a model that can be used in *globk*
- Concept: The departure from a white-noise (\sqrt{n}) reduction in noise with averaging provides a measure of correlated noise.
- Implementation:
 - Fit the values of χ^2 vs averaging time to the exponential function expected for a first-order Gauss-Markov (FOGM) process (amplitude, correlation time)
 - Use the χ^2 value for infinite averaging time predicted from this model to scale the white-noise sigma estimates from the original fit
 - and/or
 - Fit the values to a FOGM with infinite averaging time (i.e., random walk) and use these estimates as input to *globk* (`mar_neu` command)

Extrapolated variance (FOGMEx)

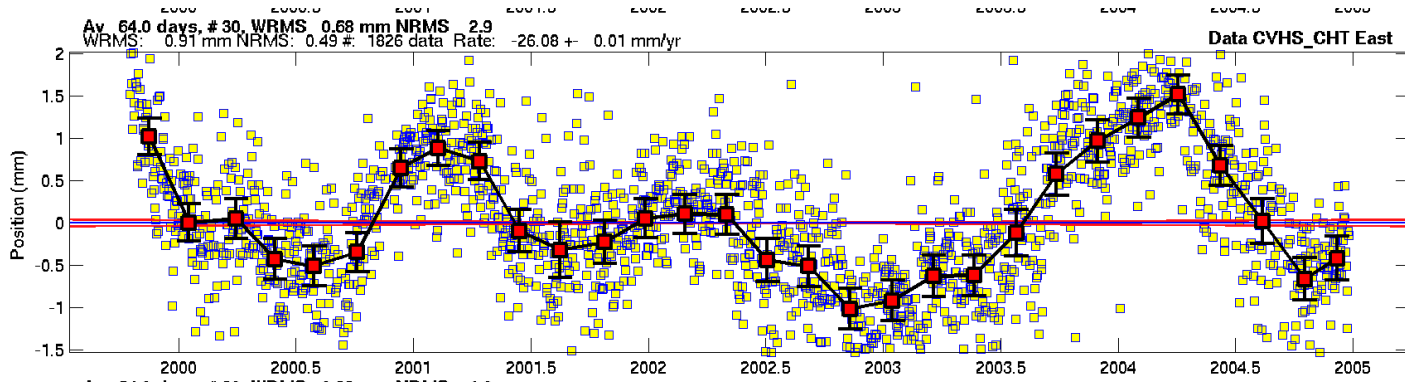
- For independent noise, variance $\propto 1/\sqrt{N_{\text{data}}}$
- For temporally correlated noise, variance (or $\chi^2/\text{d.o.f.}$) of data increases with increasing window size
- Extrapolation to “infinite time” can be achieved by fitting an asymptotic function to RMS as a function of time window
 - $\chi^2/\text{d.o.f.} \propto e^{-\sigma\tau}$
- Asymptotic value is good estimate of long-term variance factor
- Use “real_sigma” option in `tsfit`



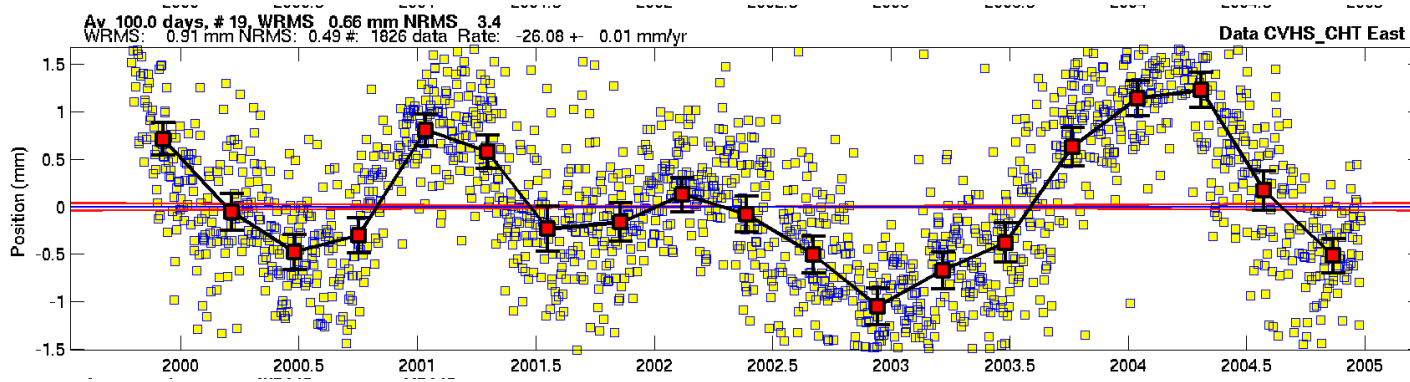
Understanding the FOGMEx algorithm: Effect of averaging on time-series noise



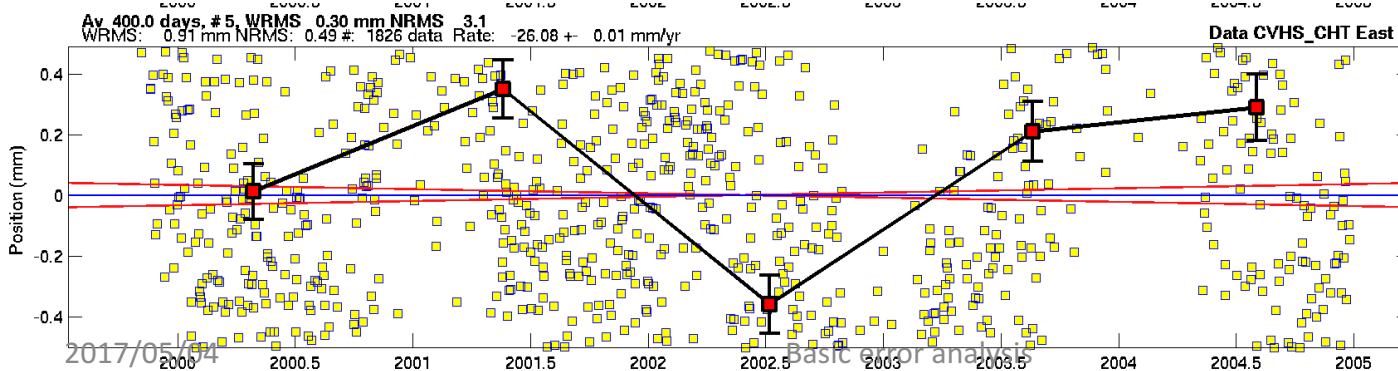
Same site, East component (daily wrms 0.9 mm nrms 0.5)



64-d avg
wrms 0.7 mm
nrms 2.0

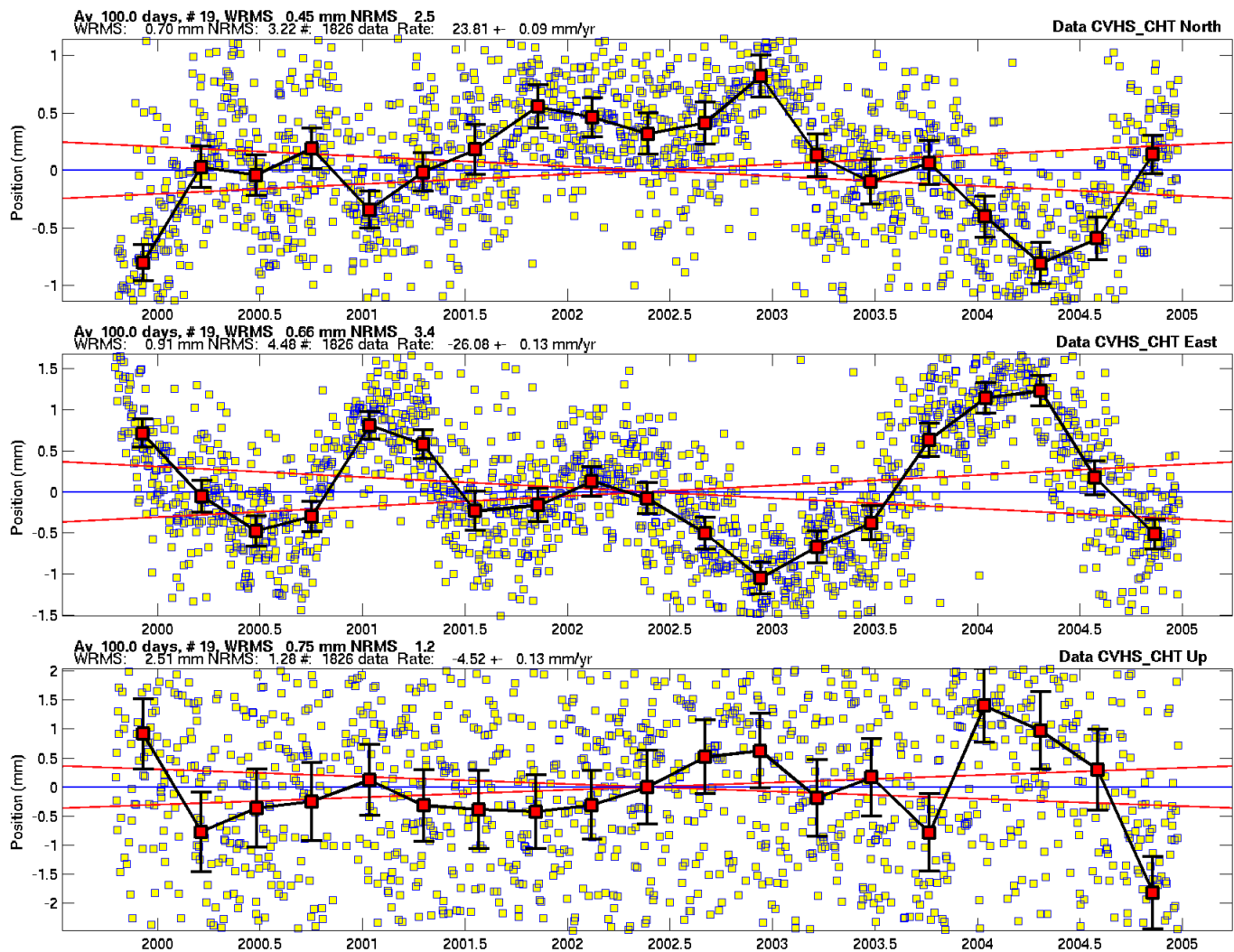


100-d avg
wrms 0.6 mm
nrms 3.4



400-d avg
wrms 0.3 mm
nrms 3.1

Using TSVIEW to compute and display the “realistic-sigma” results

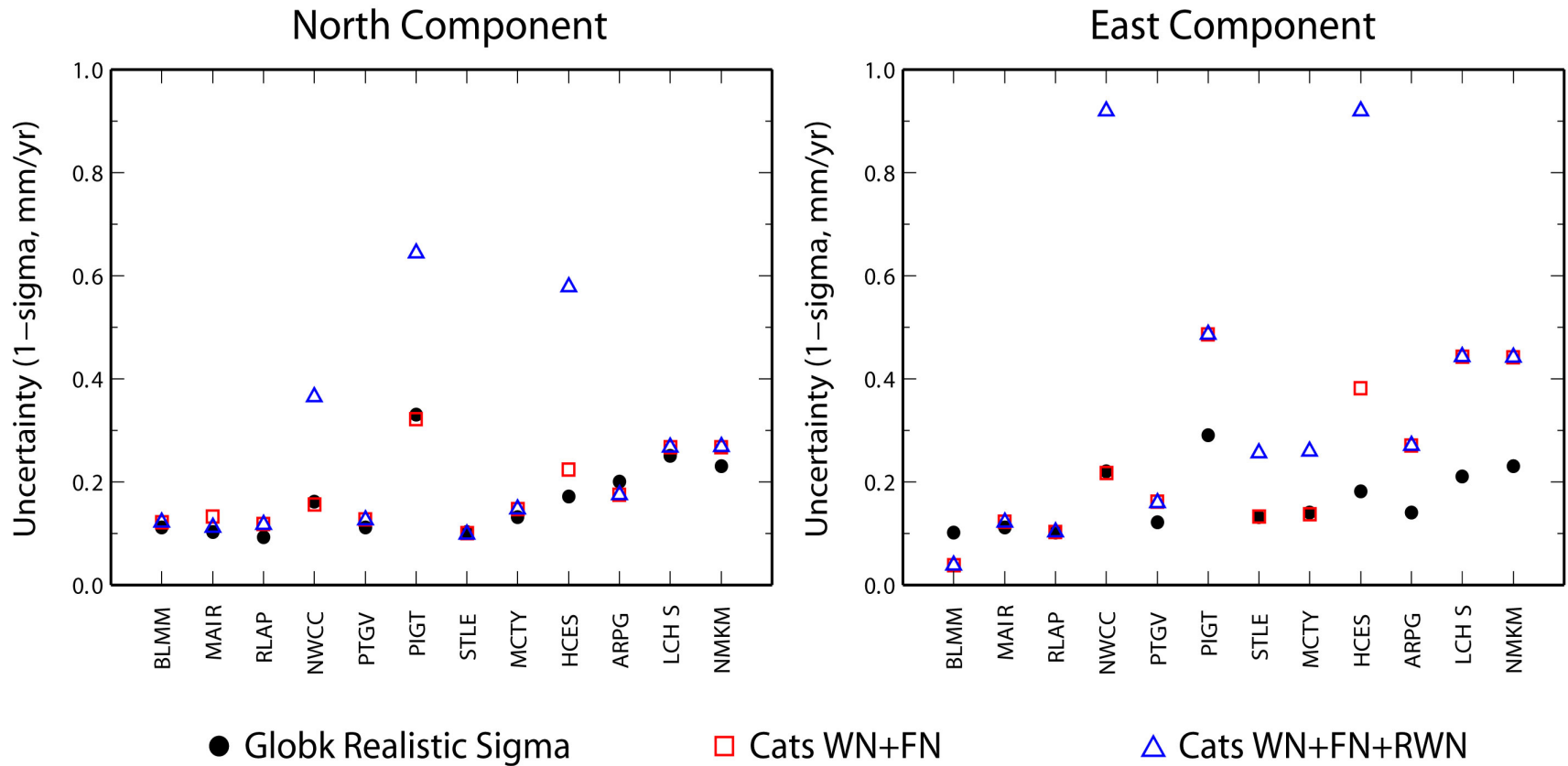


Note rate uncertainties with the “realistic-sigma” algorithm :

0.09 mm/yr N
0.13 mm/yr E
0.13 mm/yr U

Red lines show the 68% probability bounds of the velocity based on the results of applying the algorithm.

Comparison of estimated velocity uncertainties using spectral analysis (CATS) and Gauss-Markov fitting of averages (FOGMEx)



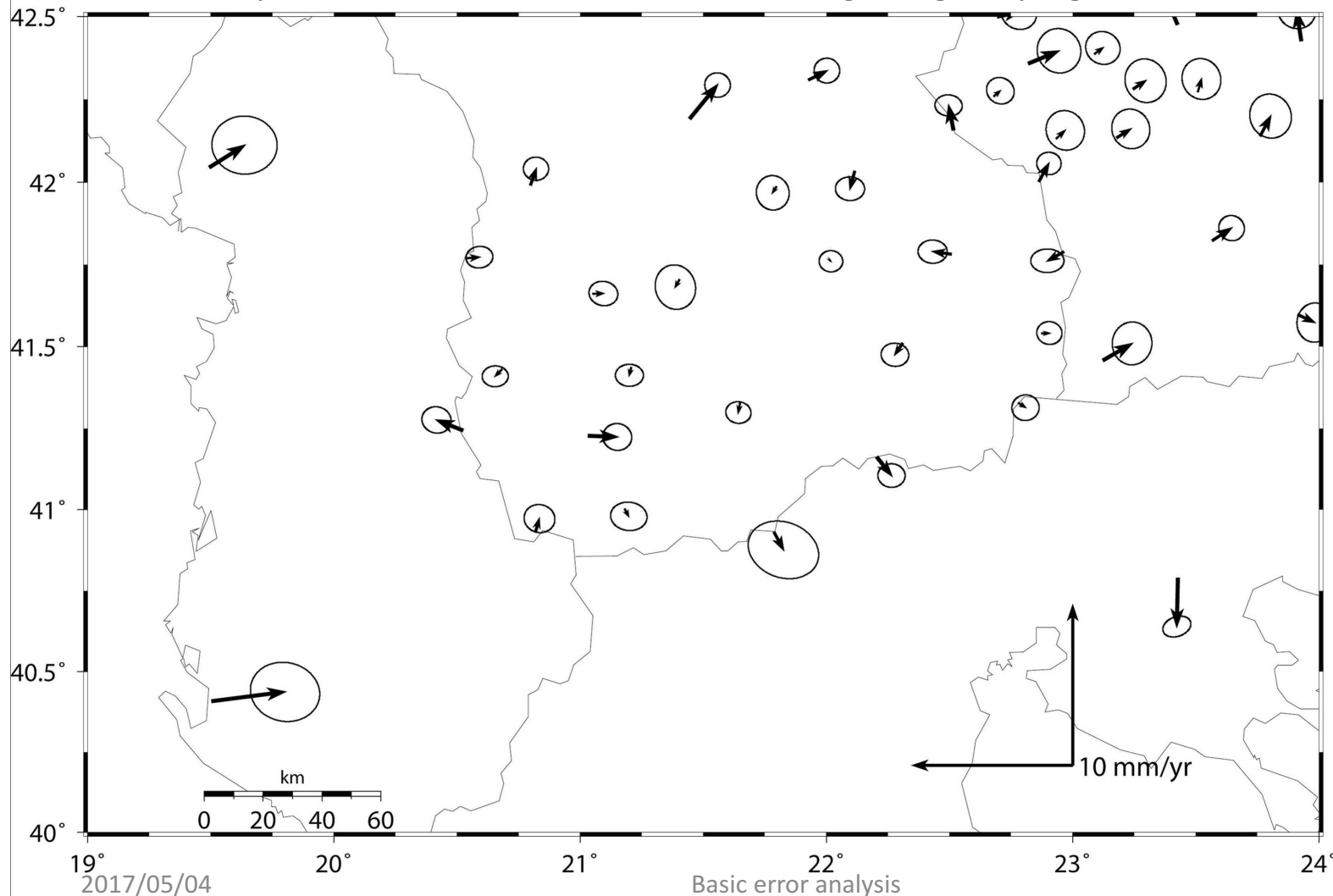
Plot courtesy E. Calais

Summary of Practical Approaches

- White noise + flicker noise (+ random walk) to model the spectrum [Williams et al., 2004]
 - White noise as a proxy for flicker noise [Mao et al., 1999]
 - Random walk to model to model an exponential spectrum [Herring “FOGMEx” algorithm for velocities]
 - “Eyeball” white noise + random walk for non-continuous data
-
- Only the last two can be applied in GLOBK for velocity estimation
 - All approaches require common sense and verification

External validation of velocity uncertainties by comparing with a model

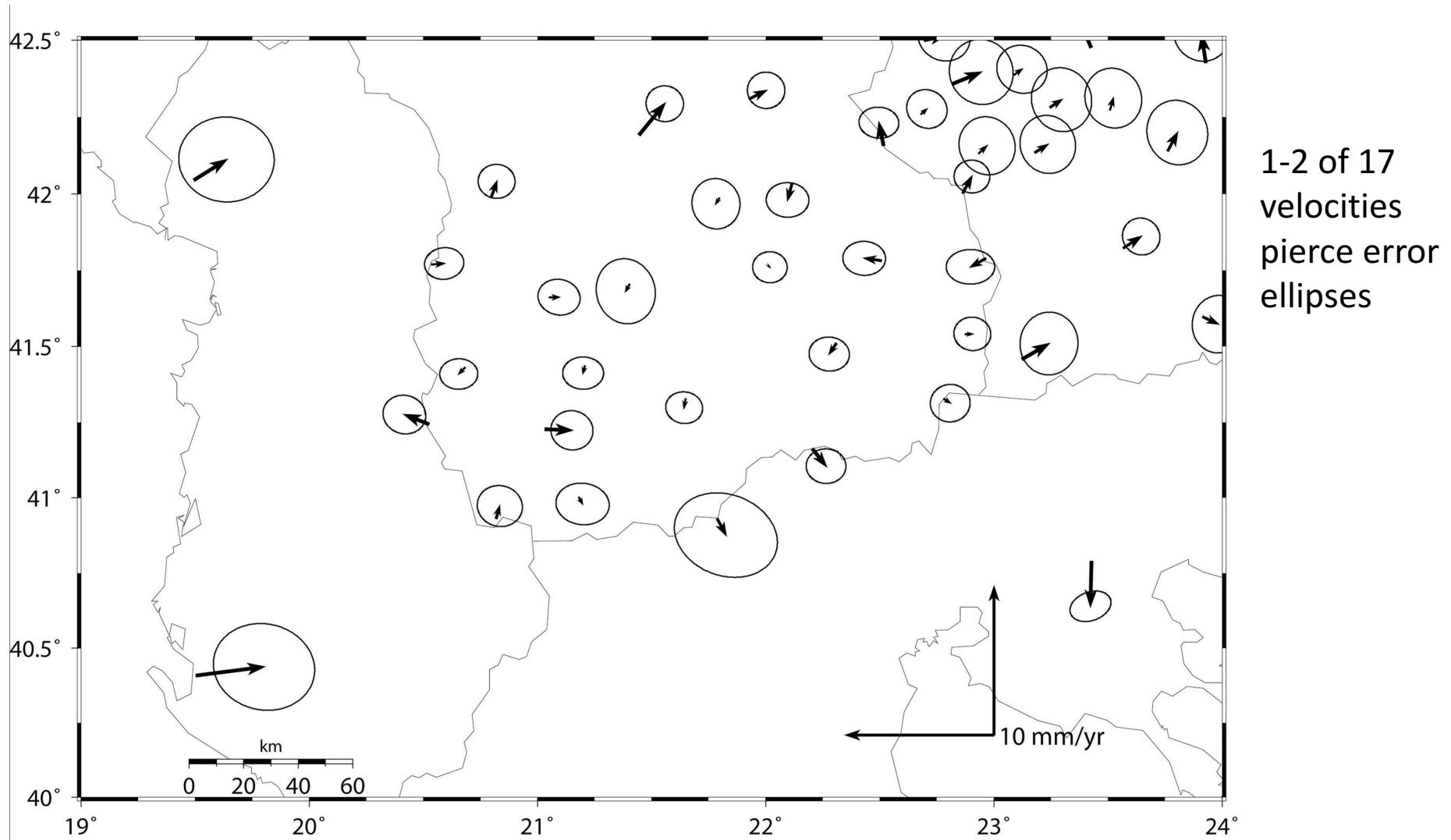
- Simple case: assume no strain within a geologically rigid block



GMT plot at 70% confidence

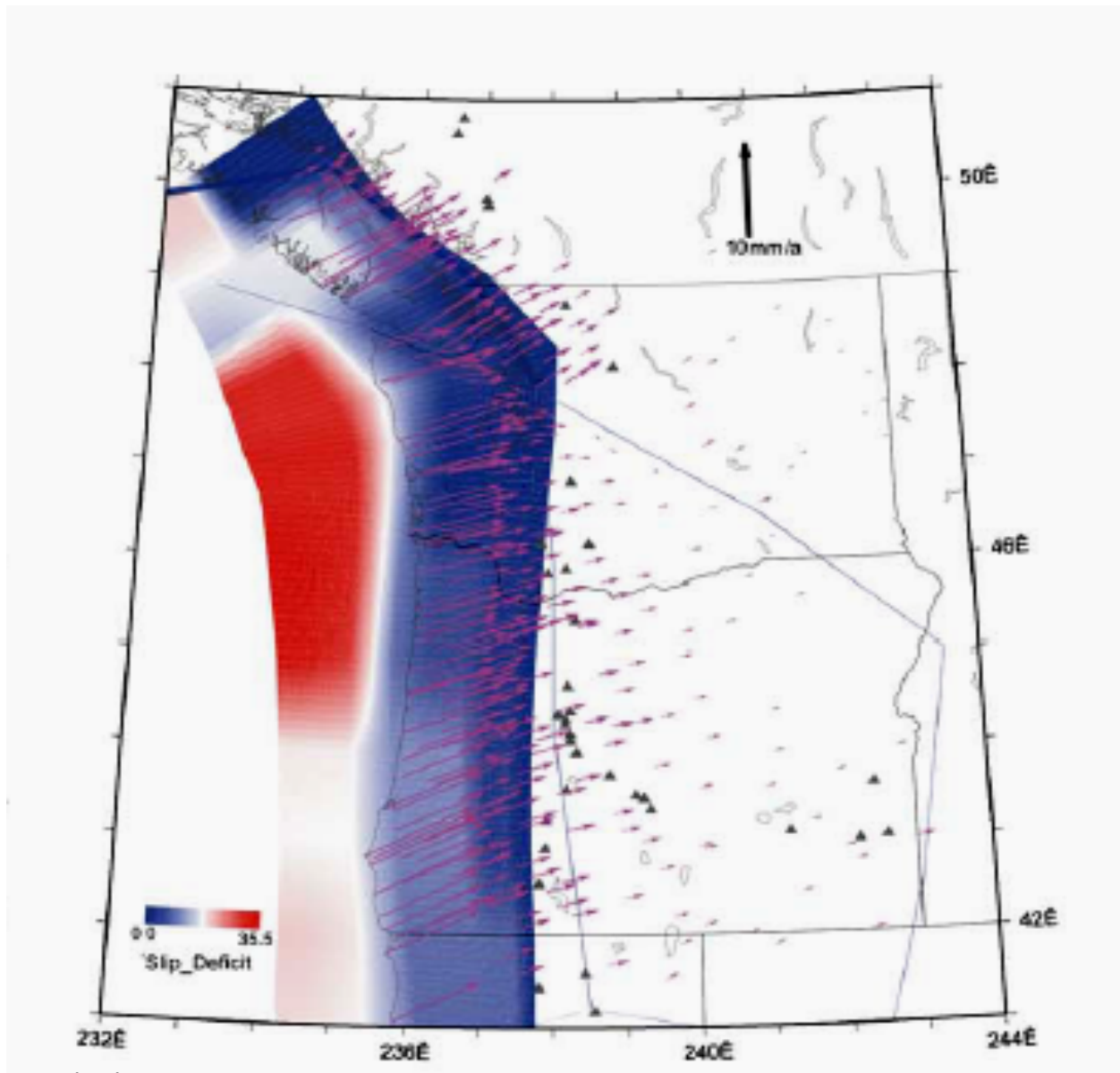
17 sites in central Macedonia: 4-5 velocities pierce error ellipses

.. same solution plotted with 95% confidence ellipses

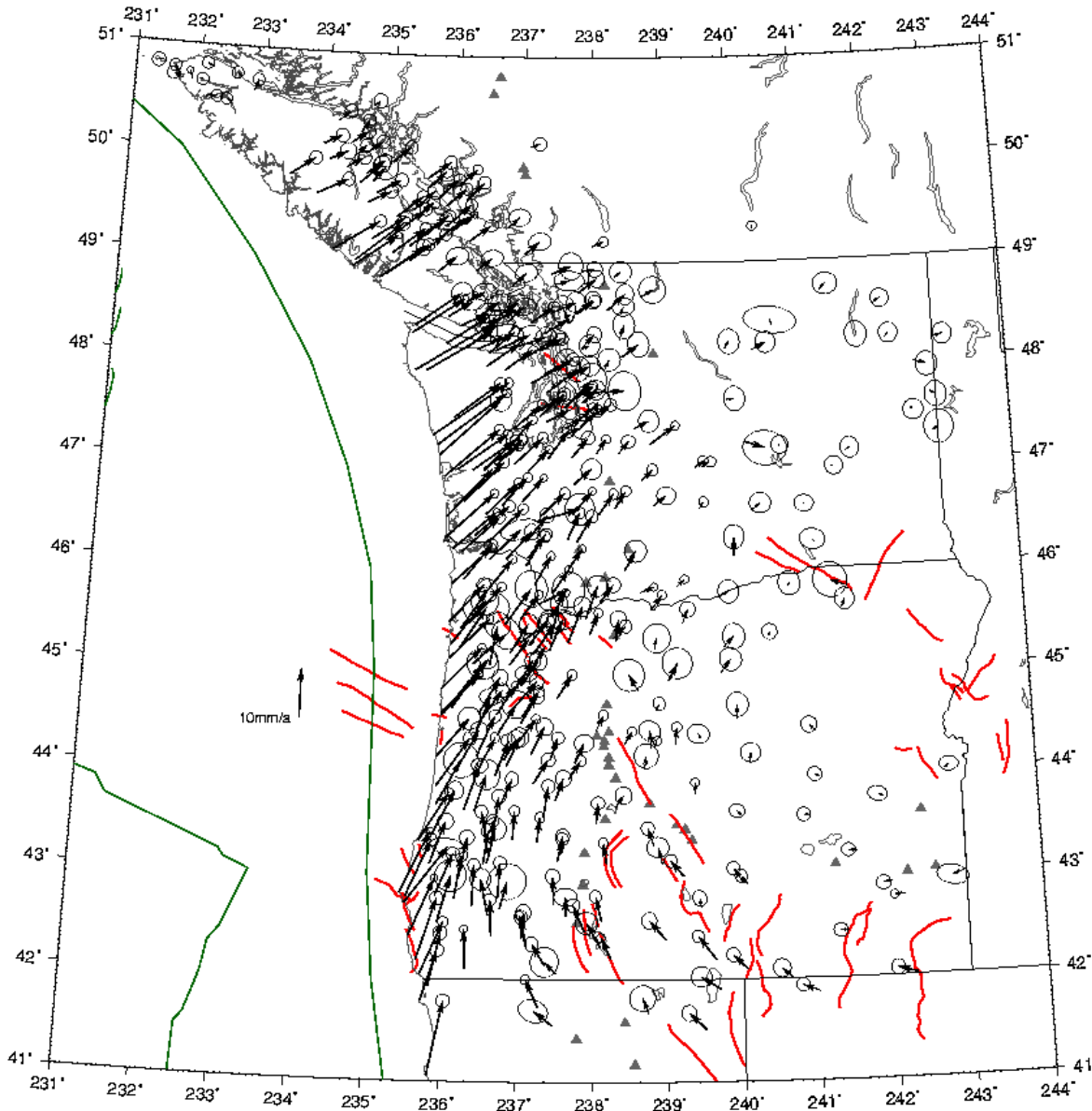


External validation of velocity uncertainties by comparing with a model

- a more complex case of a large network in the Cascadia subduction zone

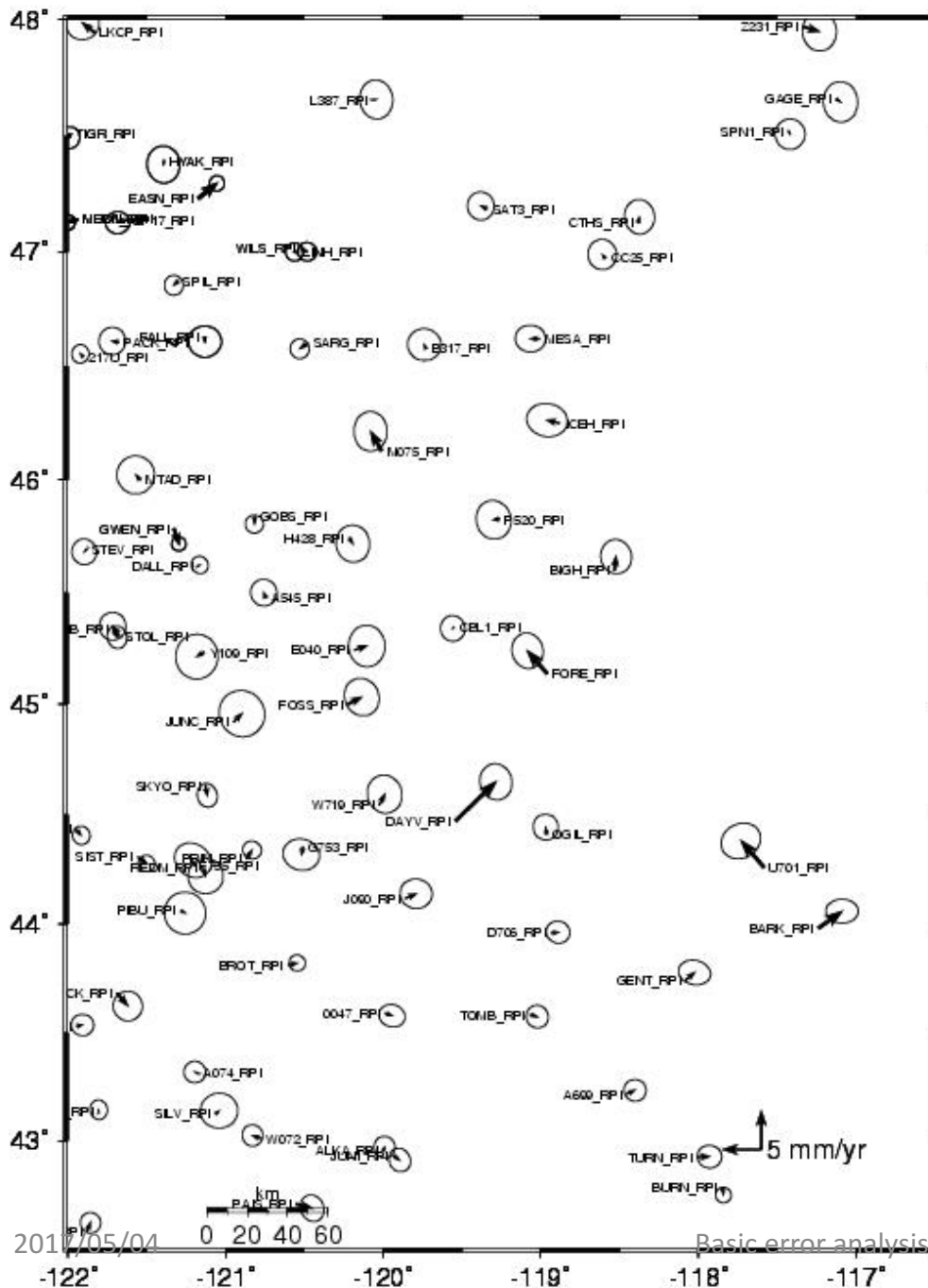


Colors show slipping and locked portions of the subducting slab where the surface velocities are highly sensitive to the model; area to the east is slowly deforming and insensitive to the details of the model



Velocities and
70% error
ellipses for 300
sites observed by
continuous and
survey-mode
GPS 1991-2004

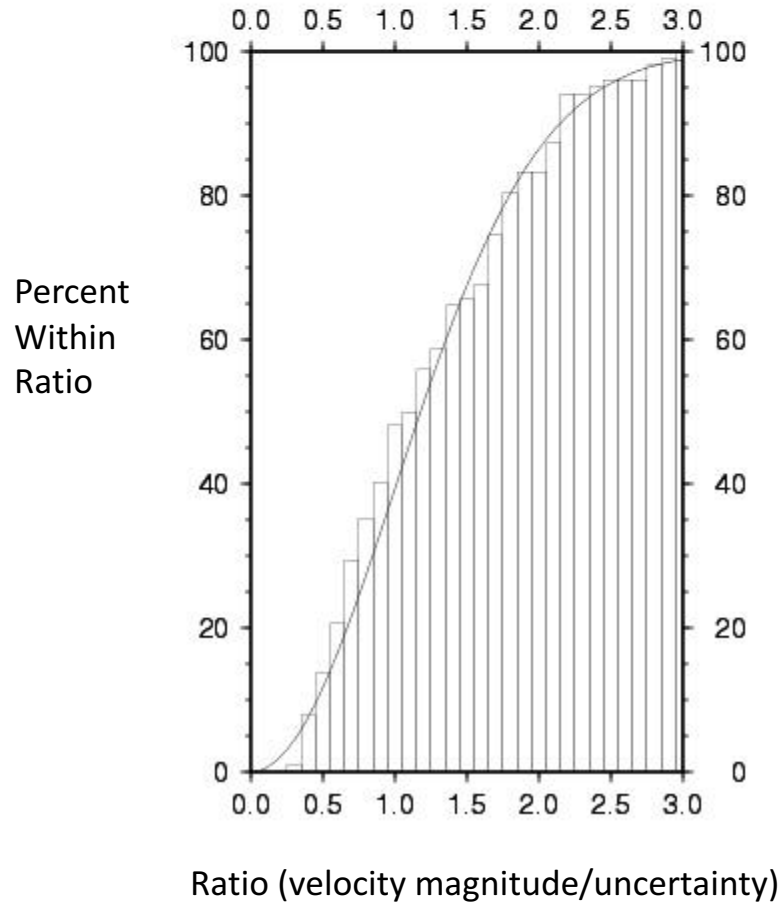
Test area (next
slide) is east of
238E



Residuals to elastic block model for 73 sites in slowly deforming region

Error ellipses are for 70% confidence:
 13-17 velocities pierce their ellipse

Statistics of Velocity Residuals

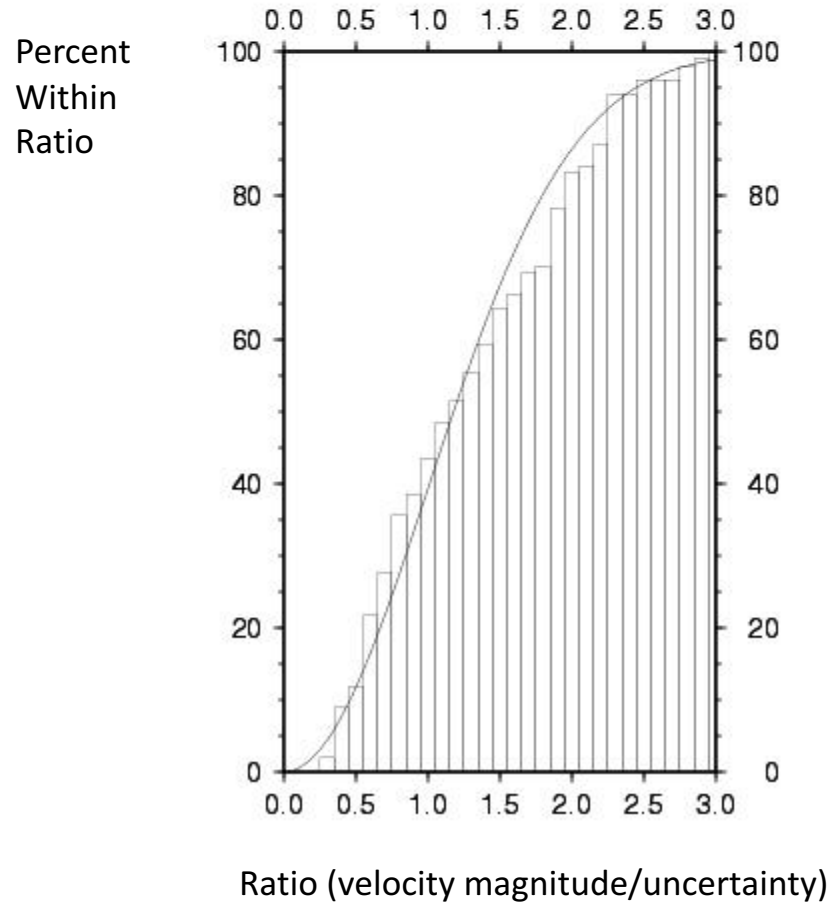


Cumulative histogram of normalized velocity residuals for Eastern Oregon & Washington (70 sites)

Noise added to position for each survey:
0.5 mm random
1.0 mm/sqrt(yr) random walk

Solid line is theoretical for a chi distribution

Statistics of Velocity Residuals



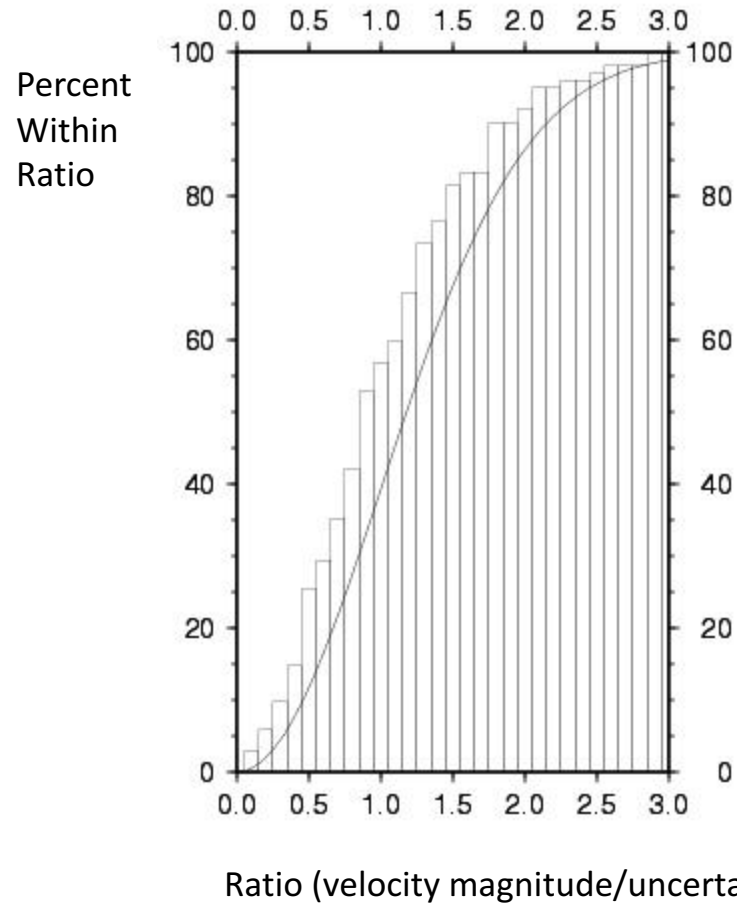
Same as last slide but with a smaller random-walk noise added :

0.5 mm random
0.5 mm/yr random walk

(vs 1.0 mm/sqrt(yr)) RW for
'best' noise model)

Note greater number of
residuals in range of 1.5-2.0
sigma

Statistics of Velocity Residuals



Same as last slide but with larger random and random-walk noise added :

2.0 mm white noise
1.5 mm/sqrt(yr) random walk

(vs 0.5 mm WN and 1.0 mm/sqrt(yr))
RW for 'best' noise model)

Note smaller number of residuals in all ranges above 0.1-sigma

Summary

- All algorithms for computing estimates of standard deviations have various problems: Fundamentally, rate standard deviations are dependent on low frequency part of noise spectrum which is poorly determined.
- Assumptions of stationarity are often not valid
- FOGMEx (“realistic sigma”) algorithm is a convenient and reliable approach to getting velocity uncertainties in *globk*
- Velocity residuals from a physical model, together with their uncertainties, can be used to validate the error model

References

Spectral Analysis

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Realistic Sigma Algorithm

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