



# Reference frames

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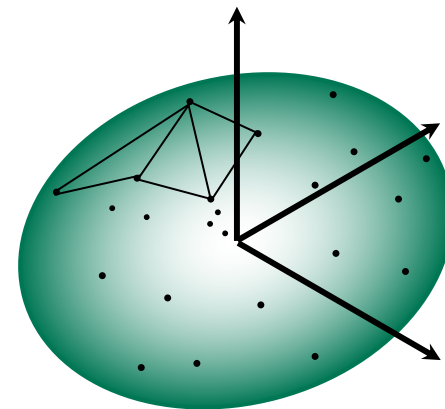
24–28 August 2020

[http://geoweb.mit.edu/~floyd/courses/gg/202008\\_UNAVCO/](http://geoweb.mit.edu/~floyd/courses/gg/202008_UNAVCO/)

Material from R. W. King, T. A. Herring, M. A. Floyd (MIT) and S. C. McClusky (now at ANU)

# Reference frames

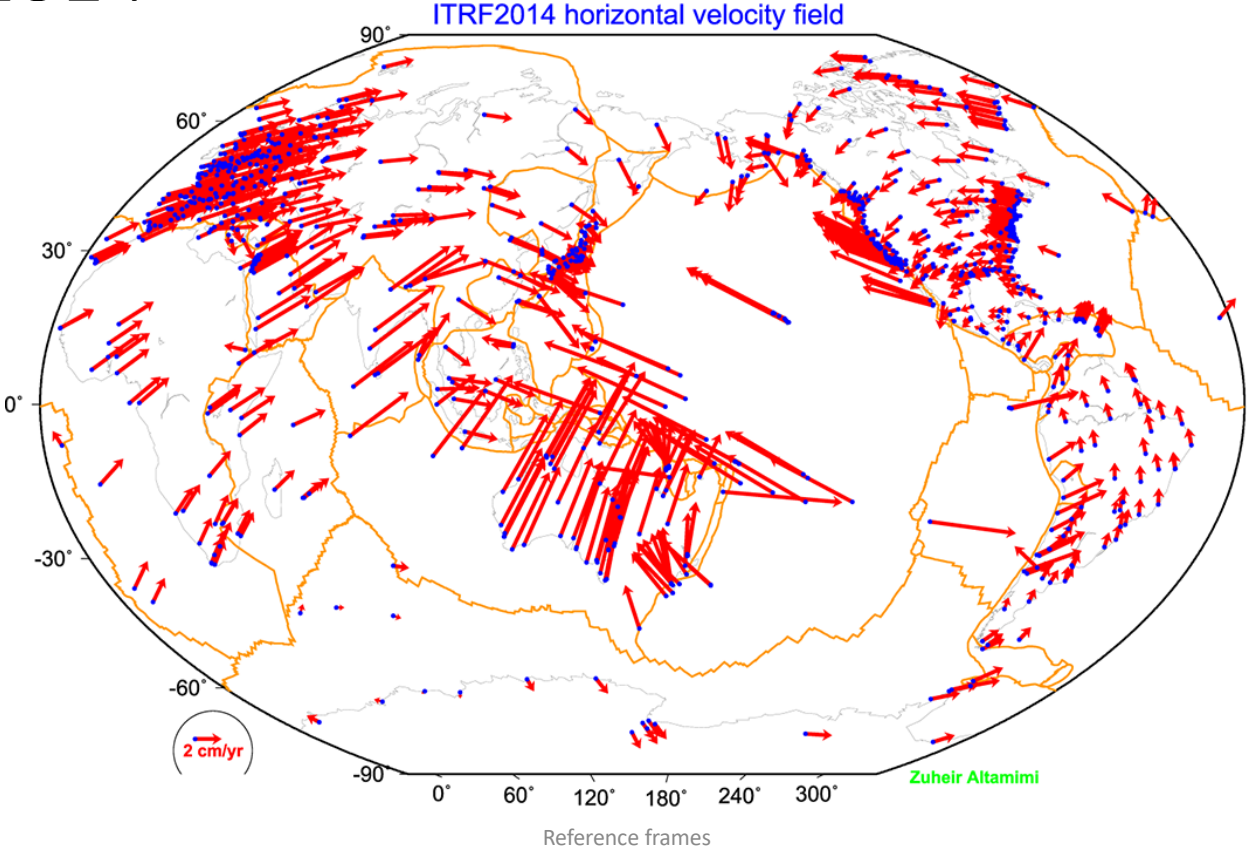
- Physical realization of reference system
- Requires fixed measurement sites which are defined by position and velocity in the reference frame and thus constitute realizing the system
  - VLBI
  - SLR
  - GNSS
- Non-linear motions are now also included.



# Ultimately, everything is relative

- Even “absolute positioning” is relative to some coordinate origin, orientation and scale
- So what is “global positioning”?
- The center and rotation pole of Earth define our coordinate axes
  - But how do we know where the center of the Earth is?
  - Do we really care?
    - It depends on your goals
- No-net-rotation criterion (surface integral of angular velocity is zero) is usually used as the constraint on defining a terrestrial reference frame in the presence of surface motions (plate tectonics)

# ITRF2014



# ITRF2014

What are some general features of plate motion that you can see?

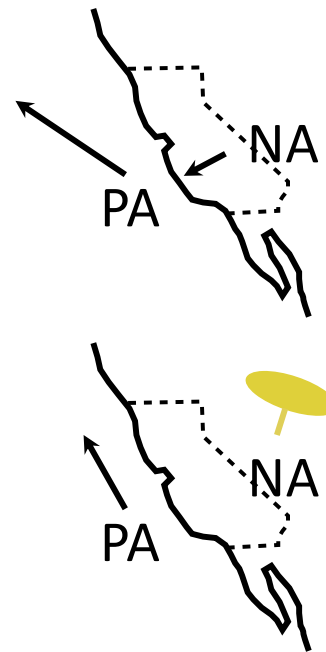
- North America rotates around a point in the Pacific off South America
- Eurasia and Africa appear to have very similar motions
- Antarctica is moving very little
  
- ITRF2014 is a not-net-rotation frame which is a mathematical construction. For some geophysical problems other frames may make more sense (e.g., mantle fixed, hot spot frame, specific plate).
- Frames for different plates (based on Euler pole fits) are in [~/gg/tables](#).

# Choices of reference frame

- Choose your reference frame based on your geophysical objectives
  - Velocities in ITRF are difficult to interpret visually from a geophysical perspective
    - Local surroundings of a volcano
    - One side of a fault
    - Upper plate of a subduction zone
- Major plate reference frame
  - Major plates are often chosen to conform with conventional perspectives of velocity solutions
  - Relative to Eurasia, Nubia, North America, South America, etc.
  - But don't feel restricted by this. Sometimes your geophysical discussion is best visualized relative to any stable boundary of a deforming region
- Regional reference frame
  - Central Valley of California, non-deforming part of Anatolia, smaller coherent regions, etc.
- Local reference frame
  - Sites near but outside the influence of a volcano, geothermal field, etc.

# Examples

- Expressing velocities in ITRF is not very meaningful or useful when we want to look at the deformation at a plate boundary, e.g. the San Andreas Fault system
- Better to look at velocities with one side “fixed” so we can see what the other side is doing relative to it



# Basic issues in reference frame realization

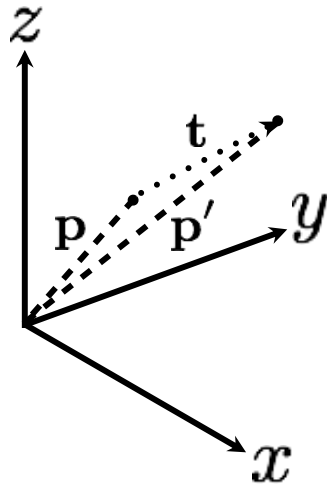
- Concept is to align the estimated site positions and possibly velocity to a set of well defined locations that have physical significance for the analysis being performed (e.g., GAGE aligns to a realization of the North America plate based on ITRF2014)
- `glorg` is the module which does this and computes the covariance matrix of the aligned solution in the reference frame chosen.
- Transformation is often called an  $N$ -parameter Helmert transformation
  - $N = 3$ : translation only (could also be just rotation)
  - $N = 6$ : translation and rotation
  - $N = 7$ : translation, rotation and scale
- In GLOBK analyses, you need to decide
  - How many parameters (3/6/7)
  - Sites to use to determine the parameters (`sh_gen_stats`)
  - Values of the positions/velocities of the reference frame sites
  - Weight to be given to heights in computing the transformation parameters ("`cnd_hgtv`" command; first two arguments for position and velocity, other arguments are sigma limits)



# Reference frame implementation

- Any vector may be mapped from one coordinate system to another by the application of
  - Translation (affects position of co-ordinate origin)
  - Rotation (affects orientation of co-ordinate axes)
  - Scale (affects length of co-ordinate axes and maps into heights in the network). It is important to note that scale changes are just changes to the average height differences in the network. It is very common to estimate scale changes but be aware that this impacts estimates of height changes.

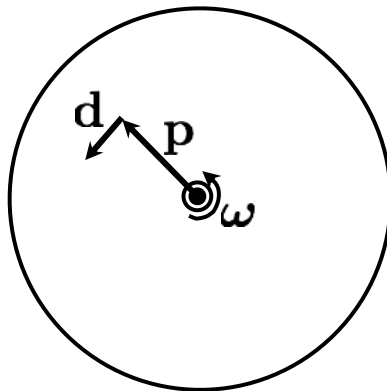
# 1: Translation



$$\mathbf{p}' = \mathbf{p} + \mathbf{t}$$
$$\begin{pmatrix} p'_x \\ p'_y \\ p'_z \end{pmatrix} = \begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \\ t_z \end{pmatrix}$$

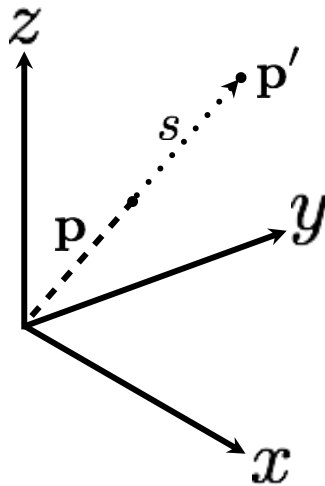
## 2: Rotation

- Rotation vector,  $\boldsymbol{\omega}$ , is defined as in the direction of the rotation axis with length equal to the magnitude of angular rotation
- Displacement (or velocity) vector then makes a right-handed triplet with the rotation vector,  $\boldsymbol{\omega}$ , and radial vector,  $\mathbf{p}$ .



$$\begin{aligned}\mathbf{d} &= \boldsymbol{\omega} \times \mathbf{p} \\ &= \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \times \begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix} \\ \mathbf{d} &= \mathbf{R}\mathbf{p} \\ &= \begin{pmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{pmatrix} \begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix}\end{aligned}$$

### 3: Scale



$$\mathbf{p}' = \mathbf{p} + s\mathbf{p}$$
$$\begin{pmatrix} p'_x \\ p'_y \\ p'_z \end{pmatrix} = \begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix} + s \begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix}$$

Scale estimation has a direct effect on the average height adjustments in the network so be cautious if your interested in height changes over the region of your reference frame definition.

# Helmert transformation

- Position

$$\mathbf{p}' = \mathbf{p} + \mathbf{t} + \mathbf{R}\mathbf{p} + s\mathbf{p}$$

$$\begin{pmatrix} p'_x \\ p'_y \\ p'_z \end{pmatrix} = \begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \\ t_z \end{pmatrix} + \begin{pmatrix} s & -\omega_z & \omega_y \\ \omega_z & s & -\omega_x \\ -\omega_y & \omega_x & s \end{pmatrix} \begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix}$$

- Velocity

$$\mathbf{v}' = \mathbf{v} + \dot{\mathbf{t}} + \cancel{\mathbf{R}\mathbf{v}} + \dot{\mathbf{R}}\mathbf{p} + \cancel{s\mathbf{v}} + \dot{s}\mathbf{p}$$

$$\begin{pmatrix} v'_x \\ v'_y \\ v'_z \end{pmatrix} = \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} + \begin{pmatrix} \dot{t}_x \\ \dot{t}_y \\ \dot{t}_z \end{pmatrix} + \begin{pmatrix} s & -\omega_z & \omega_y \\ \omega_z & s & -\omega_x \\ -\omega_y & \omega_x & s \end{pmatrix} \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} + \begin{pmatrix} \dot{s} & -\dot{\omega}_z & \dot{\omega}_y \\ \dot{\omega}_z & \dot{s} & -\dot{\omega}_x \\ -\dot{\omega}_y & \dot{\omega}_x & \dot{s} \end{pmatrix} \begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix}$$

- Usually, the terms  $\mathbf{R}\mathbf{v}$  and  $s\mathbf{v}$  are very small and can be neglected ( $< 10^{-6}$  rad  $\times < 0.1$  m/yr and  $< 0.1$  ppb  $\times < 0.1$  m/yr, respectively)

# Reference frames in geodetic analyses

- Output from GAMIT
  - Loosely constrained solutions
  - Relative position well determined, “absolute” position weakly defined
  - Need a procedure to express coordinates in a well defined reference frame
- Two aspects
  - Theoretical (e.g. rigid block, mantle-fixed, no-net-rotation of plates)
  - Realization through a set of coordinates and velocities
    - “Finite constraints”: a priori sigmas on site coordinates
    - “Generalized constraints”: minimize coordinate residuals while adjusting translation, rotation, and scale parameters
- Three considerations in data processing and analysis
  - Consistent with GNSS orbits and EOP (NNR)
    - not an issue if network small or if orbits and EOP estimated
  - Physically meaningful frame in which to visualize site motions
  - Robust realization for velocities and/or time series

# Frame definition with finite constraints

- Applied in `globk` (`glorg` not called)
- We do not recommend this approach since it is sensitive to over-constraints that can distort velocities and positions but some small networks (e.g., volcanoes) this may be easiest approach.

- Example:

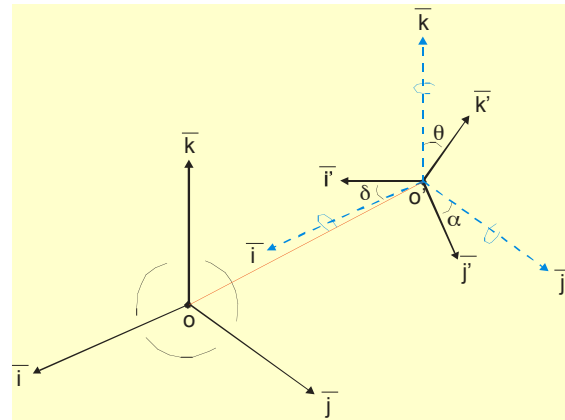
```
apr_file igb14_comb.apr
apr_neu all 10 10 10 1 1 1
apr_neu algo .005 .005 .010 .001 .001 .003
apr_neu piel .005 .005 .010 .001 .001 .003
apr_neu drao .005 .005 .010 .002 .002 .005
...
```

- Most useful when only one or two reference sites or very local area
- Disadvantage for large networks is that bad a priori coordinates or bad data from a reference site can distort the network

# Frame definition with generalized constraints

- Applied in `glorg`: minimize residuals of reference sites while estimating translation, rotation, and/or scale (3–7 parameters)

```
apr_file igb14_comb.apr
pos_org xtran ytran ztran xrot yrot zrot
stab_site algo piel drao ...
cnd_hgtv 10 10 0.8 3.
stab_it 4 0.5 2.5
```



- All reference coordinates free to adjust (anomalies more apparent), outliers are iteratively removed by `glorg`
- Network can translate and rotate but not distort (no strain except rotational)
- Works best with strong redundancy (number and [if rotation] geometry of coordinates exceeds number of parameters)



# Defining a reference frame with glorg

- Create an .apr-file for use by glorg
  - Apply known rotation rate to .apr-file (e.g. igb14\_comb.apr → igb14\_comb\_eura.apr)
  - Zero velocity .apr-file records (and iterate using sh\_exg1k to create updated .apr-file)
- Define set of sites (must be included in GAMIT processing or other H-file input to GLOBK) which define stable region
  - “plate” in glorg command file can be used to estimate Euler pole of user defined plate.

# Stabilization using a global set of sites

- Use 40 or more sites with good velocities determined in the ITRF frame
- The `igb14_comb.apr` file, when used together with `igb14_comb.eq` to account consistently for instrumental changes over time, provides the widest choice of sites, 1992–2020
- Combining your solution with the MIT SINEX files (available from CDDIS) which have 350-sites each day.
  - You need just 4–6 common sites (more is better), which should be of high quality but need not be well known in ITRF2014 since these “tie” sites do not need to be in your frame-realization list. (Use `-a` option in `htoglb` and estimate EOP).
- For global ITRF stabilization, you can use the hierarchical list `igb14_hierarchy.stab_site` in `gg/tables`
- Although a global frame may be a convenient way to do the stabilization, it is usually not necessary for regional studies.

# Use of Global SINEX and GLX files

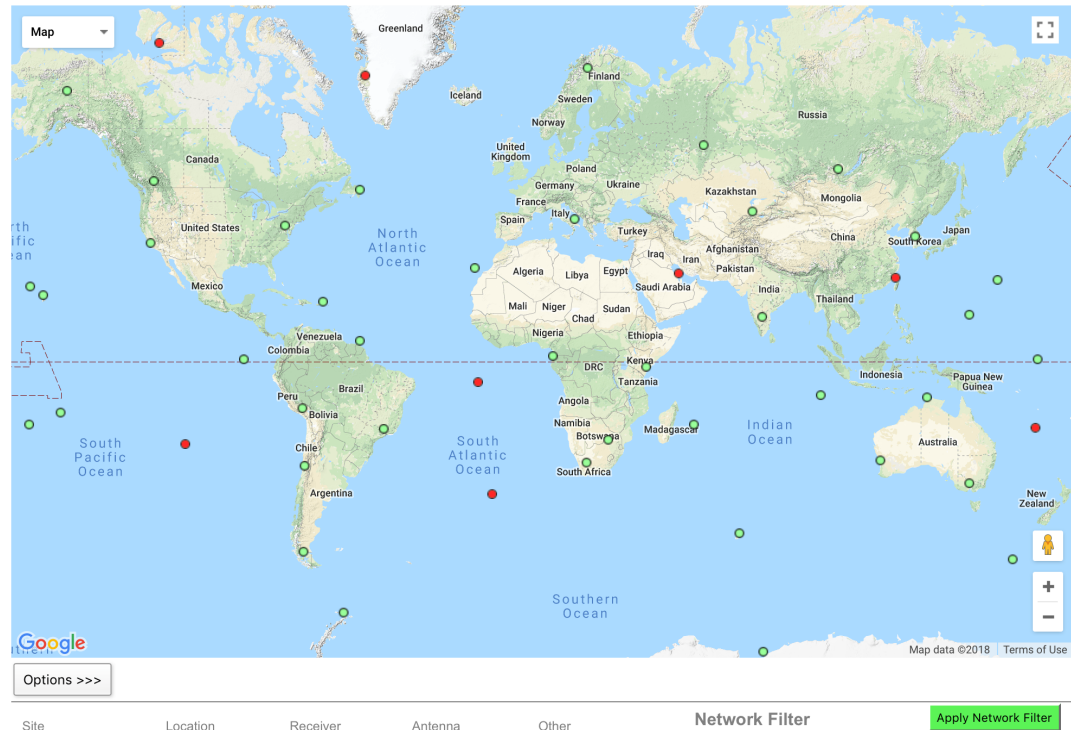
- Advantages
  - Access to a large number of sites for frame definition
  - Can (should) allow adjustment to EOP (SINEX) and orbits (GLX files)
  - Eases computational burden
- Disadvantages
  - Must use (mostly) the same models as the global processing. Less so with SINEX files.
  - For GLX file: Radiation model parameters (loose in GLX files) should be treated carefully.
  - Some bad data may be included in global h-files (can remove)
  - Greater data storage burden
- MIT global GLX available at [ftp://everest.mit.edu/pub/MIT\\_GLL/H<YY>](ftp://everest.mit.edu/pub/MIT_GLL/H<YY>)  
When using MIT files, add “apr\_svant all F F F” to globk command file to fix the satellite antenna offsets
- MIT SINEX files: <ftp://cddis.gsfc.nasa.gov/gps/products/<week>> where <week> is GPS week (program doy for conversions). (SINEX PCO constrained  $\pm 1$ mm.)

# Stabilization using regional or local sites

- If your area of study has a robust cGNSS network (10 or more well-distributed sites) with accurate a priori velocities, then `g1org` stabilization is robust and little thought is involved (`g1org` will automatically discard the one or two sites which may be weak or inconsistent)
- If your region is short on cGNSS stations with well-known coordinates, you will need to think carefully about the choice of sites to include in your solution and use the initial stabilization. A stabilization site should have
  - High quality data over the full span of your study
  - Coordinates well-known in ITRF
  - Provide symmetric coverage around your study area (except that if the region is small enough, a translation-only stabilization may be possible and distribution is less important)

# IGS (IGS14) reference frame core network

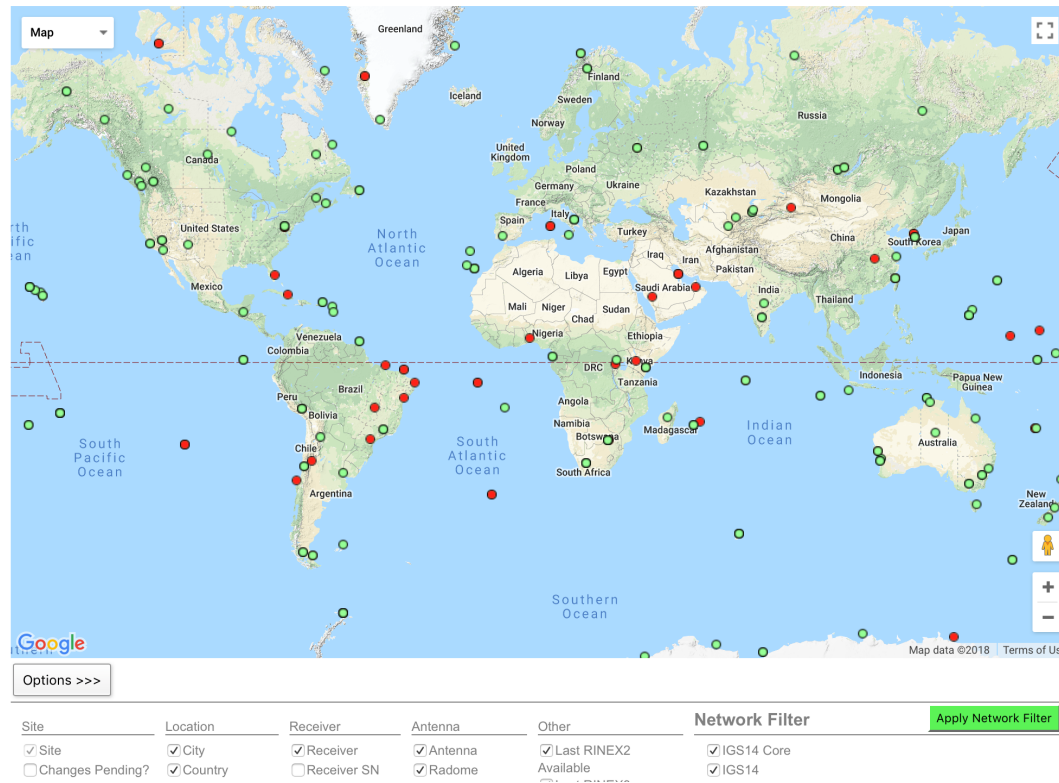
- DRAO
- GOLD
- GUAT
- KOKB
- MDO1
- MKEA
- NLIB
- PETP
- SCH2
- SCUB
- STJO
- USNO
- WHIT



<http://www.igs.org/network> Select IGS14 Core

Reference frames

# IGS (IGS14) reference frame network



# Reference frame implementation

- The main point of reference frame definition is to use sites with well known coordinates to stabilize your processed network
  - If your processed network already contains sites with known accurate coordinates, e.g. in `~/gg/tables/igb14_comb.apr`, then there is no need to add additional sites to your processing
  - Your “stab\_site” list in `glogr` command file should be guided by and closely associated with the contents of an accurate .apr-file, e.g. `igb14_comb.apr` by default but could be any accurate .apr-file you choose include in `globk` and `glogr` command files
    - `~/gg/tables/igb14_hierarchy.stab_site` relates to `~/gg/tables/igb14_comb.apr`
- Decide early in your processing whether or not you will need to include additional, e.g. IGS, sites in your GAMIT processing to use ultimately to realize your preferred reference frame
  - The first decision you must make (choosing sites to process) is greatly influenced by the ultimate processing step (reference frame realization)!

# Rules for stabilization of time series

- Small-extent network: translation-only in `g1org`, must constrain EOP in `g1obk`
- Large-extent network: translation and rotation, must keep EOP loose in `g1obk`
- If scale estimated in `g1org`, must estimate scale in `g1obk`
- First pass for editing:
  - Adequate “stab\_site” list of stations with accurate a priori coordinates and velocities and available most days
  - Keep in mind deficiencies in the list
- Final pass for presentation, assessment and statistics
  - Robust “stab\_site” list of all/most stations in network, with coordinates and velocities determined from the final velocity solution
- System is often iterated (velocity field solution, generate time series, editing and statistics of time series; re-generate velocity field)
- If you have time series, you can test options using `tscon`



# Issues with estimating scale

- Many GNSS analyses, automatically include scale when aligning reference frames
  - Since the Earth is almost spherical, scale changes are directly proportional to the average height difference between the reference site coordinates and their a priori values
- When comparing and analyzing height changes, how scale is treated directly effects the results
  - Aspects of this issue are discussed in a white paper that is available from the GAGE analysis documentation at UNAVCO and in Herring et al. (2016)
- Scale estimates are related the mean height differences over the reference sites
  - Should this be removed or not is an open question

## More subtle issues: Important if studying regional and continental height changes

- If regional/continental scale networks, there are high correlations between all the sites in the network especially when heights are down weighted in the frame realization (cnd\_hgtv).
- Adding stations from a global solution can help considerably. Only stations in the regional network that are in the global solution are needed (i.e., no increase in the number of stations processed). The best global solution to use are from SINEX files.
- To show the impact, the next slide looks at GNSS solutions using 98 stations from the NOTA network in Alaska and the Western United States. About 3 months of data is processed and the MIT SINEX files from CDDIS are used the global solution.

# Impact of scale and of adding global network

- Median WRMS scatters of position residuals: G-GPS, E-Galileo, R-Glonass. SNX: 4 sites from MIT IGS SNX file included.

	No Scale Estimated				Scale Estimated		
Solution	N	E	U		N	E	U
	(mm)	(mm)	(mm)		(mm)	(mm)	(mm)
MIT_G	0.74	0.79	5.11		0.75	0.80	3.11
MIT_GE	0.72	0.78	4.87		0.72	0.79	3.04
MIT_GRE	0.69	0.80	4.89		0.70	0.81	2.96
SNX_G	0.72	0.78	4.54		0.72	0.77	3.15
SNX_GRE	0.69	0.78	4.54		0.69	0.81	2.92



# Velocities and time series

- The criteria for stabilization are different for velocity solutions and time series
- Velocity solutions:
  - Physical reference is important
  - Not so sensitive to station dropout (solution holds the frame together)
- Time series:
  - Physical reference is not important
  - Sensitive to station dropout
  - Best representation of the statistics of the velocity solution is stabilization using ALL the well-determined sites from the velocity solution, now in a common frame

# Referencing to a horizontal block (“plate”)

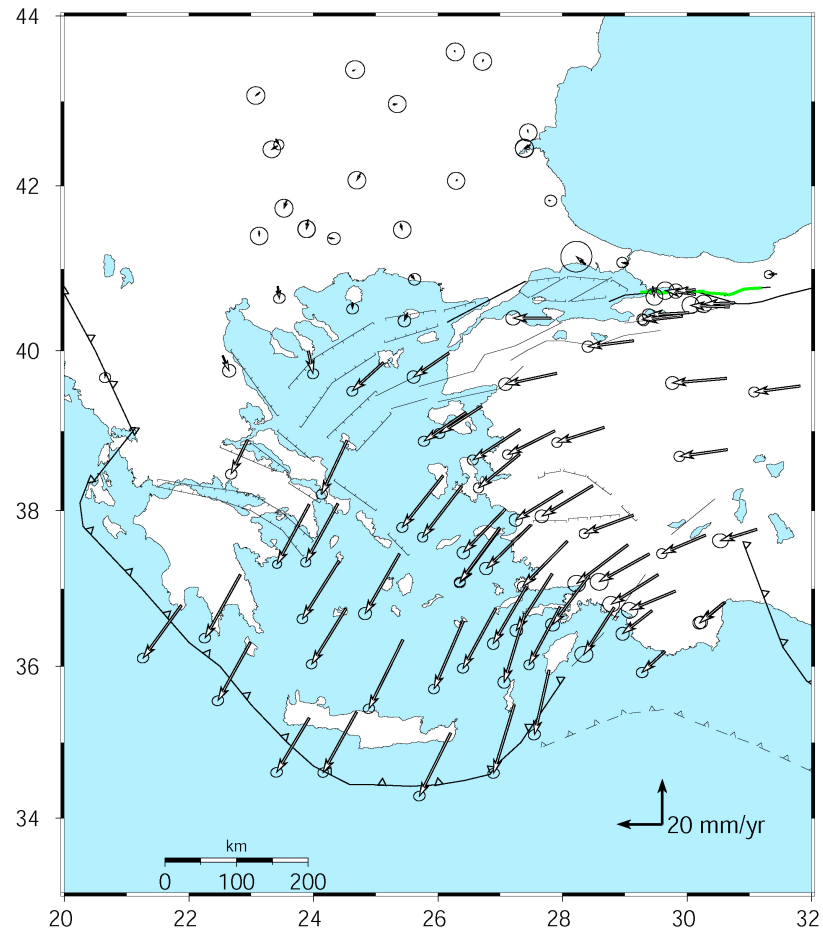
- Applied in `glorg`
- First stabilize in the usual way with respect to a reference set of coordinates and velocities (e.g. ITRF-NNR), then define one or more “rigid” blocks

```
apr_file igb14_comb.apr
pos_org xtran ytran ztran xrot yrot zrot
stab_site algo piel nlib drao gold snil mkea chat
cnd_hgtv 10 10 0.8 3.
plate noam algo piel nlib
plate pcfc snil mkea chat
```

- After stabilization, `glorg` will estimate a rotation rate vector (“Euler pole”) for each plate with respect to the frame of the full stabilization set and print the relative poles between each set of plates
- Use `sh_org2vel` to extract the velocities of all sites with respect to each plate

Example:  
Velocities of Anatolia  
and the Aegean in a  
Eurasia reference frame

Realized by minimizing the  
velocities of 12 sites over the  
whole of Eurasia  
McClusky et al. (2000)



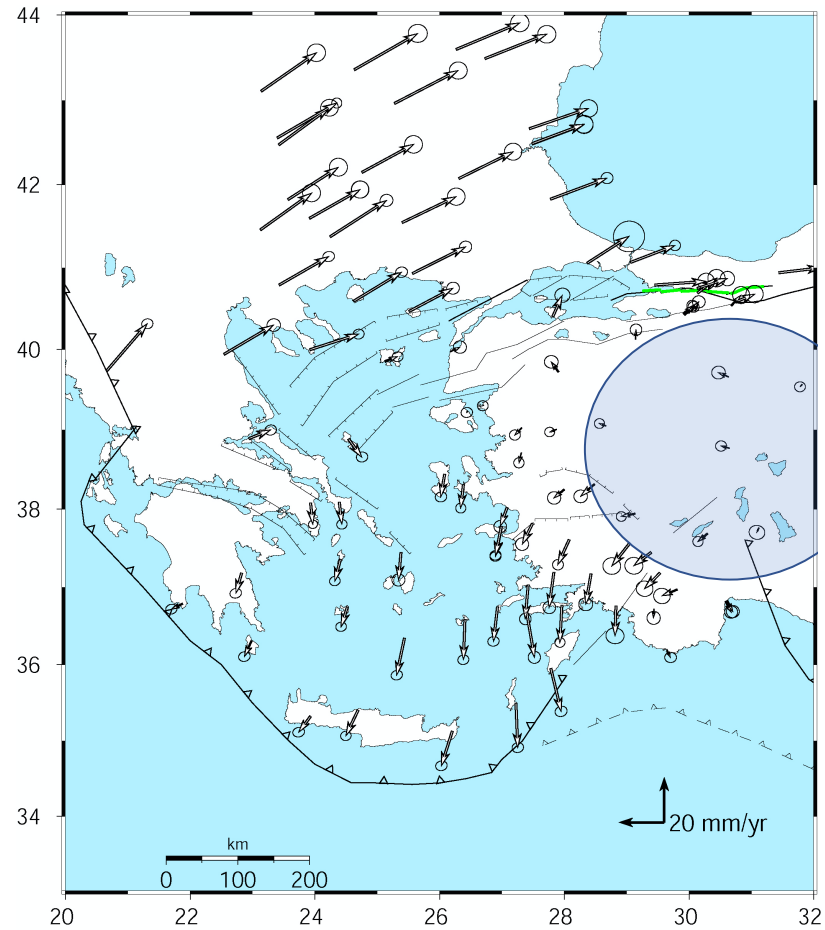
Reference frames

## Velocities in an Anatolian reference frame

Better visualization of Anatolian and Aegean deformation

Here stations in Western/Central are used to align the reference frame (a priori velocity set to zero)

McClusky et al. (2000)



# Final Thoughts

- Reference frames need to be carefully considered when results are to be displayed to best highlight the results you want to convey.
- In `g1org`: The a priori coordinate and velocity file can be changed to generate results in the desired reference frame provide the original `g1obk` analysis had loose constraints (e.g., velocity sigma  $\pm$  1 m/yr)
- Reference frames can be defined with the standard plates (e.g, `EURA_I14`, program `plate` lists all plates) or you can define your own plate using the `plate` command in `g1org` and `sh_org2vel`
- In many cases for local networks, the reference frame can be defined by zero velocities at a set of local sites
- These are the decisions that you need to make. Look in the literature to see how other groups have done this.