Error Models
Lecture 06

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Issues in GPS Error Analysis

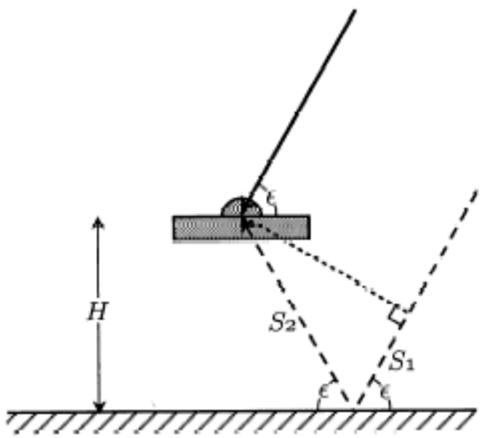
- What are the sources of the errors ?
- How much of the error can we remove by better modeling ?
- Do we have enough information to infer the uncertainties from the data ?
- What mathematical tools can we use to represent the errors and uncertainties ?

Determining the Uncertainties of GPS Parameter Estimates

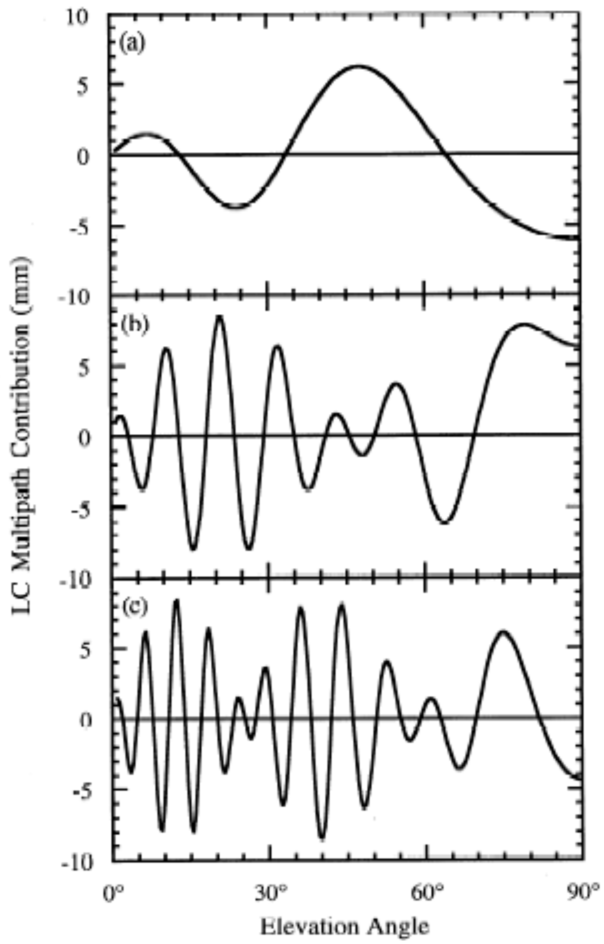
- Rigorous estimate of uncertainties requires full knowledge of the error spectrum—both temporal and spatial correlations (never possible)
- Sufficient approximations are often available by examining time series (phase and/or position) and reweighting data
- Whatever the assumed error model and tools used to implement it, external validation is important

Sources of Error

- Signal propagation effects
  - Receiver noise
  - Ionospheric effects
  - Signal scattering (antenna phase center / multipath)
  - Atmospheric delay (mainly water vapor)
- Unmodeled motions of the station
  - Monument instability
  - Loading of the crust by atmosphere, oceans, and surface water
- Unmodeled motions of the satellites

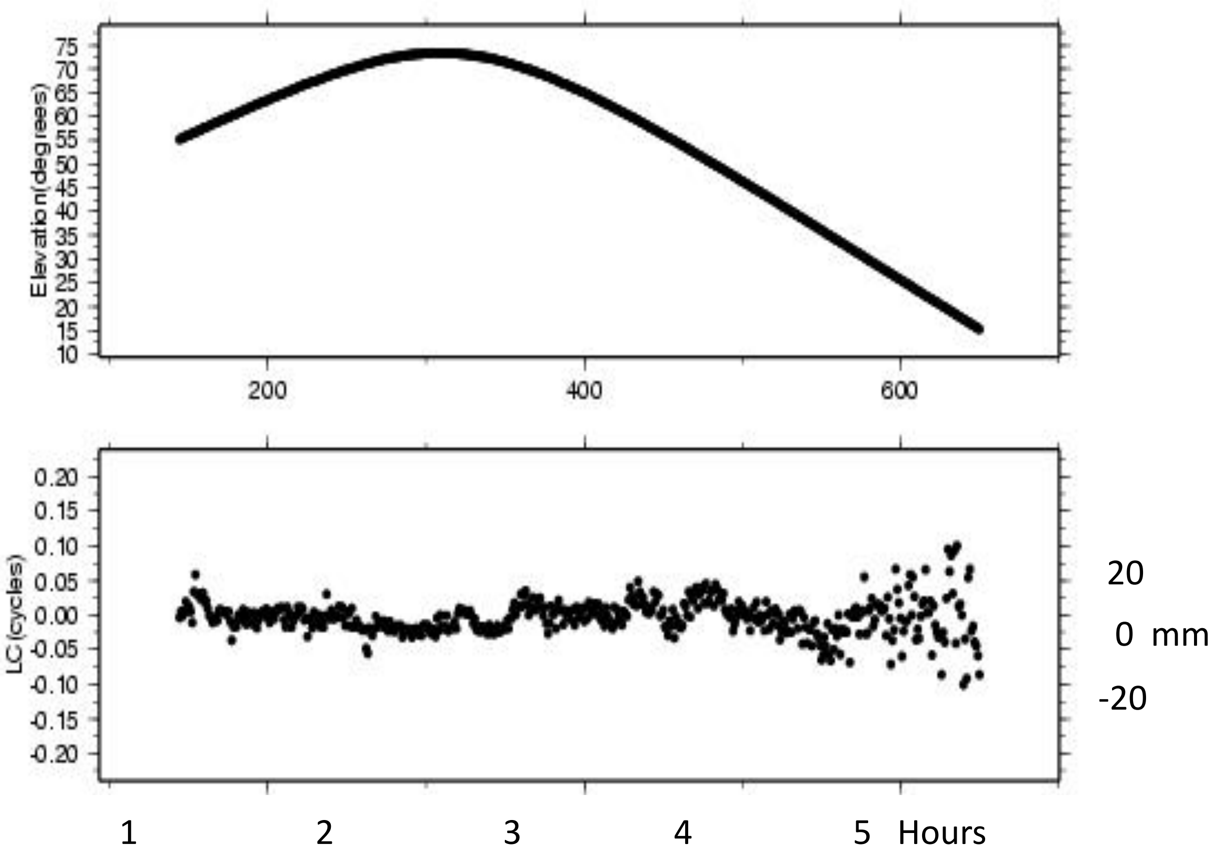


Simple geometry for incidence
of a direct and reflected signal

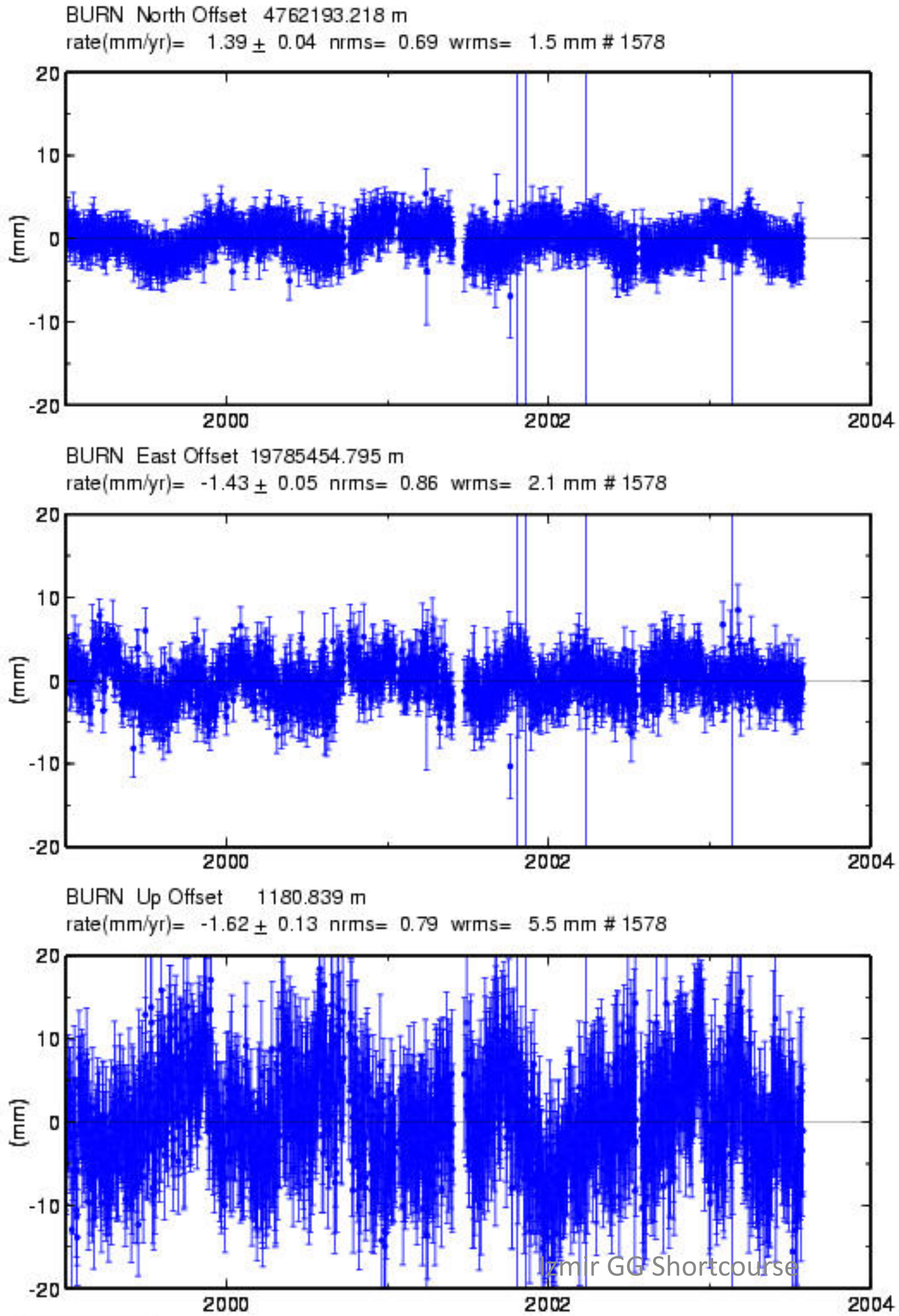


Multipath contributions to observed phase for an antenna at heights (a) 0.15 m, (b) 0.6 m, and (c) 1 m. [From *Elosegui et al*, 1995]

Characterizing Phase Noise

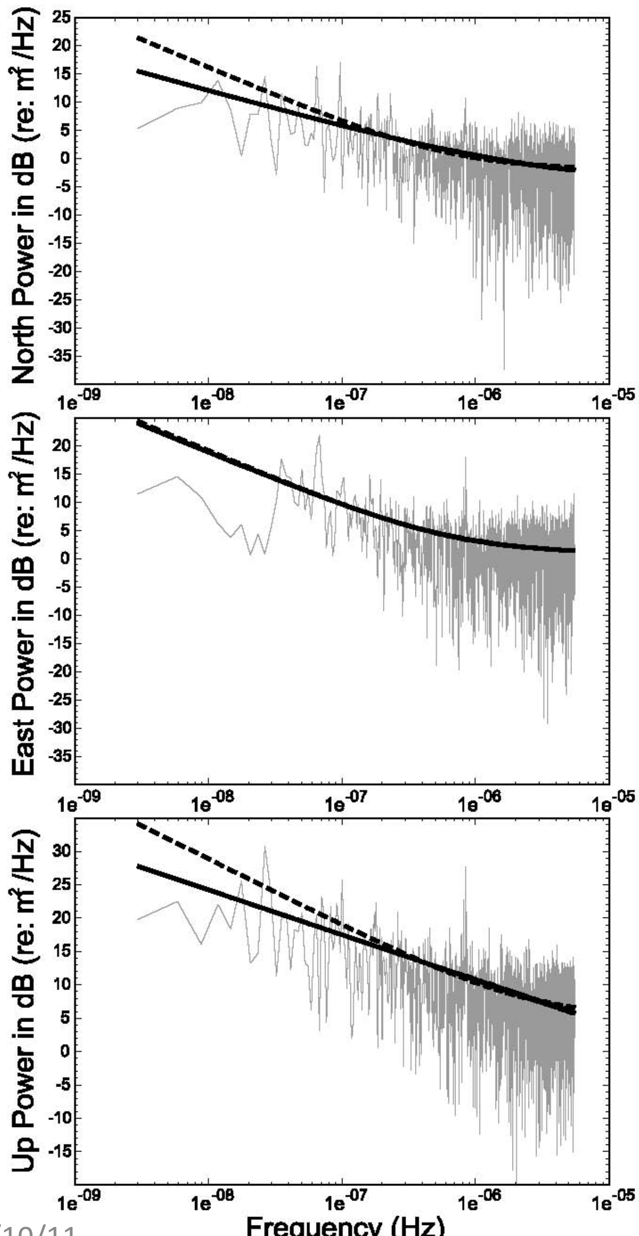


Elevation angle and phase residuals for single satellite



Characterizing the Noise in Daily Position Estimates

Note temporal correlations of 30-100 days and seasonal terms



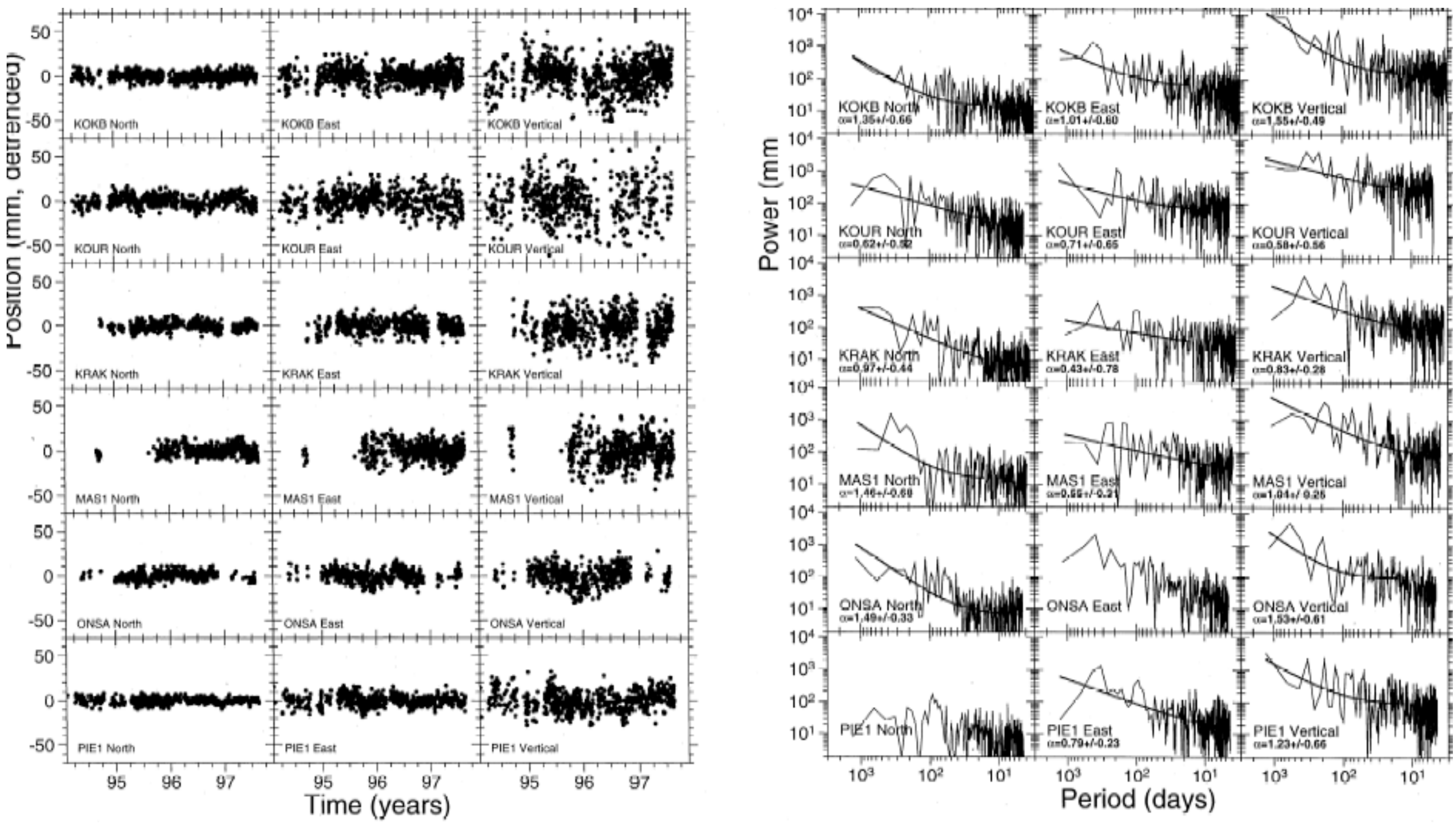
Spectral Analysis of the Time Series to Estimate an Error Model

Figure 5 from *Williams et al* [2004]: Power spectrum for common-mode error in the SOPAC regional SCIGN analysis. Lines are best-fit WN + FN models (solid=mean ampl; dashed=MLE)

Note lack of taper and misfit for periods > 1 yr

. . . spectral analysis approach

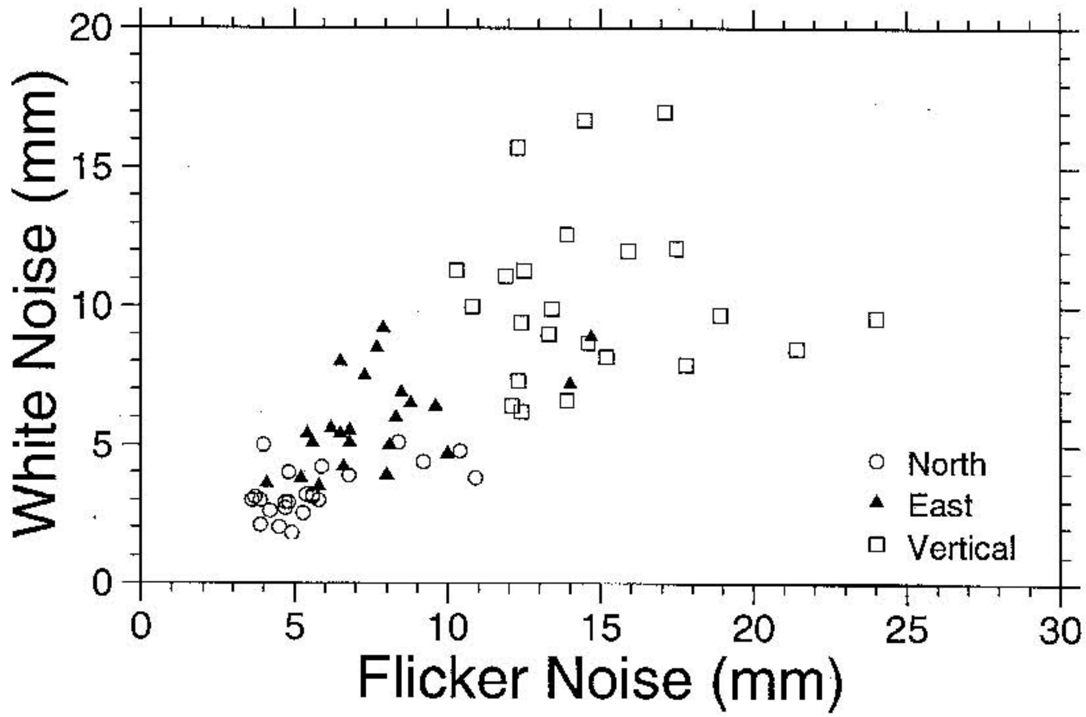
- Power law: slope of line fit to spectrum
  - 0 = white noise
  - -1 = flicker noise
  - -2 = random walk
- Non-integer spectral index (e.g. “fraction white noise” → 1 > k > -1)
- Good discussion in Williams [2003]
- Problems:
  - Computationally intensive
  - No model captures reliably the lowest-frequency part of the spectrum



Examples of times series and spectra for global stations

From *Mao et al.*, 1999

Short-cut: Use white noise statistics (wrms) to predict the flicker noise

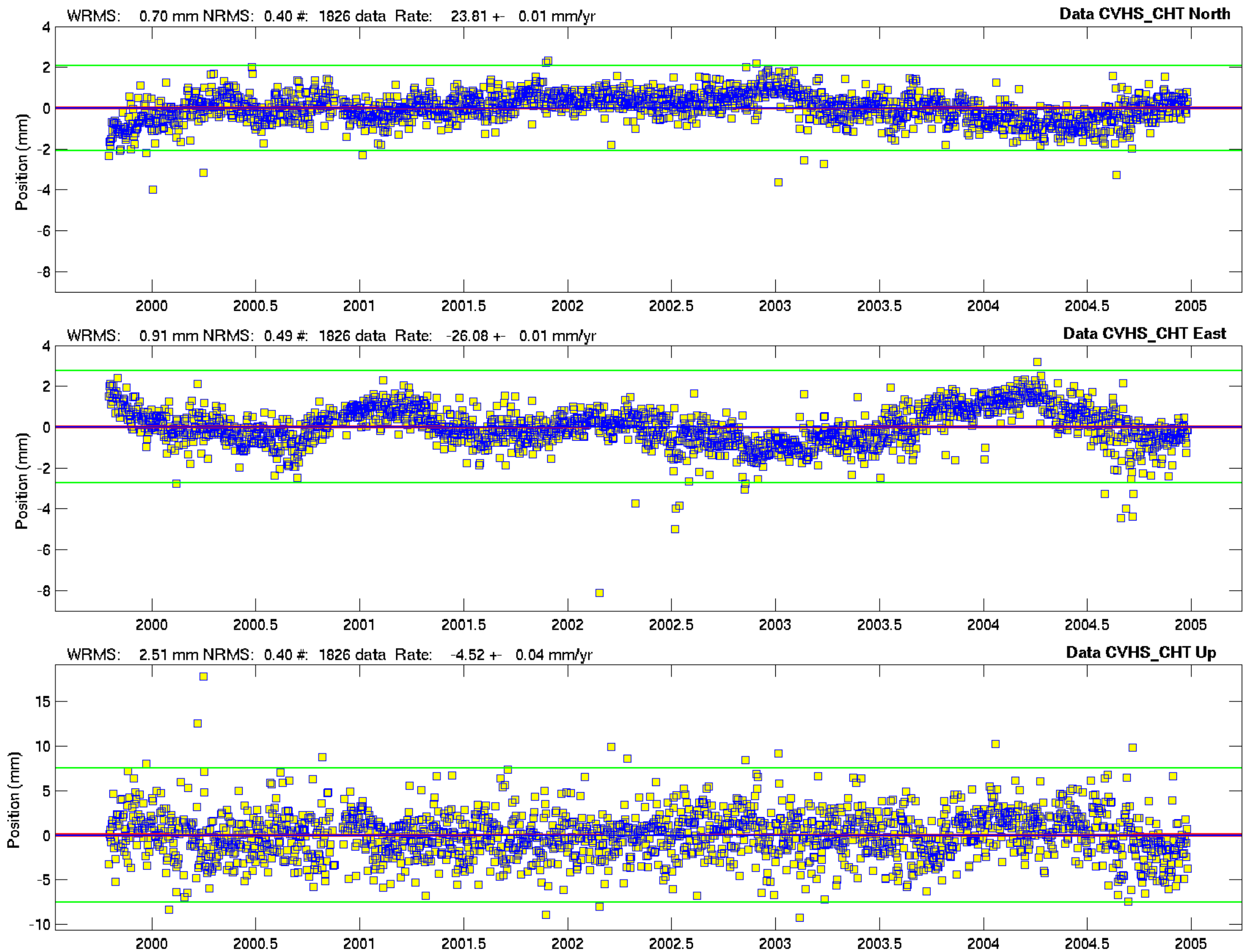


White noise vs flicker noise from *Mao et al.* [1999] spectral analysis of 23 global stations

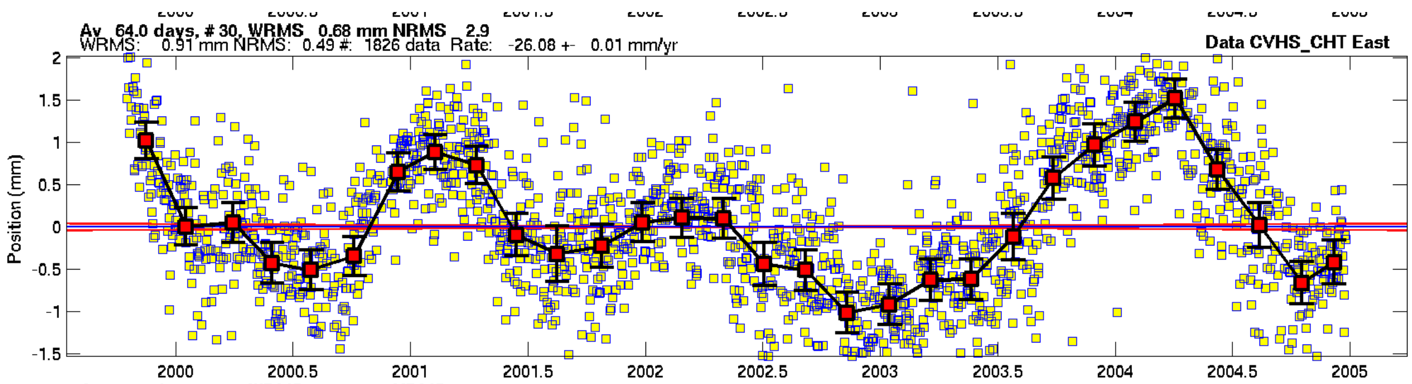
“Realistic Sigma” Algorithm for Velocity Uncertainties

- Motivation: computational efficiency, handle time series with varying lengths and data gaps; obtain a model that can be used in globk
- Concept: The departure from a white-noise (sqrt n) reduction in noise with averaging provides a measure of correlated noise.
- Implementation:
  - Fit the values of chi2 vs averaging time to the exponential function expected for a first-order Gauss-Markov (FOGM) process (amplitude, correlation time)
  - Use the chi2 value for infinite averaging time predicted from this model to scale the white-noise sigma estimates from the original fit
  - and/or
  - Fit the values to a FOGM with infinite averaging time (i.e., random walk) and use these estimates as input to globk (mar_neu command)

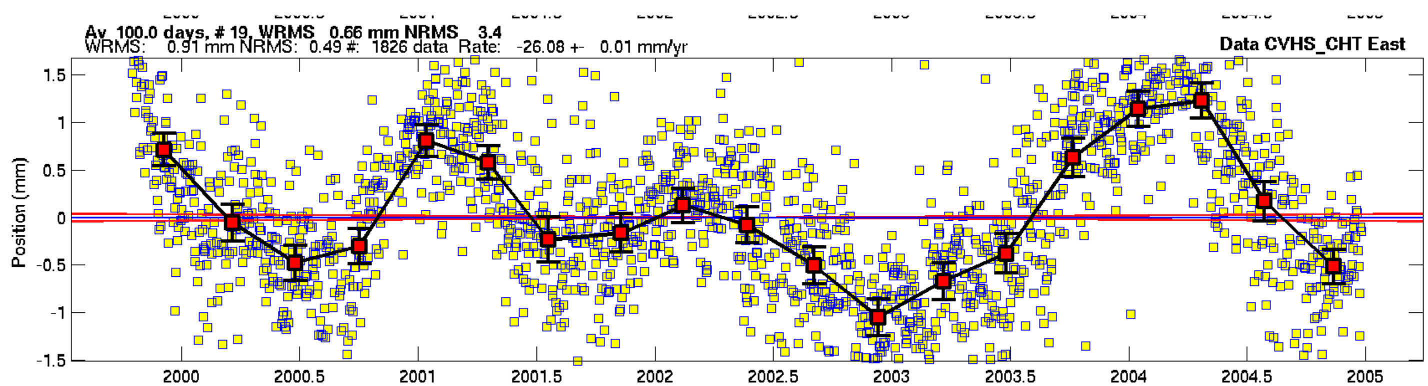
Effect of Averaging on Time-series Noise



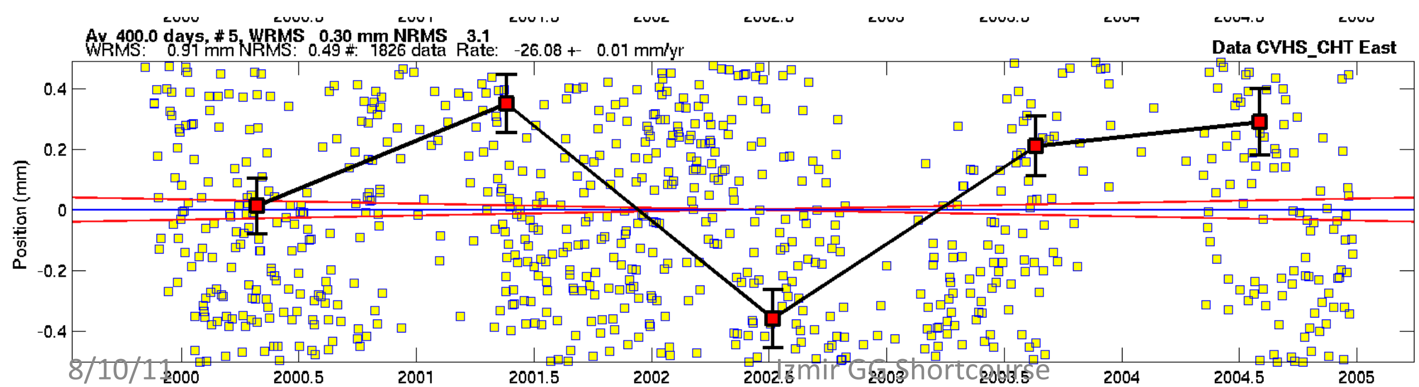
Same site, East component (daily wrms 0.9 mm nrms 0.5)



64-d avg
wrms 0.7 mm
nrms 2.0

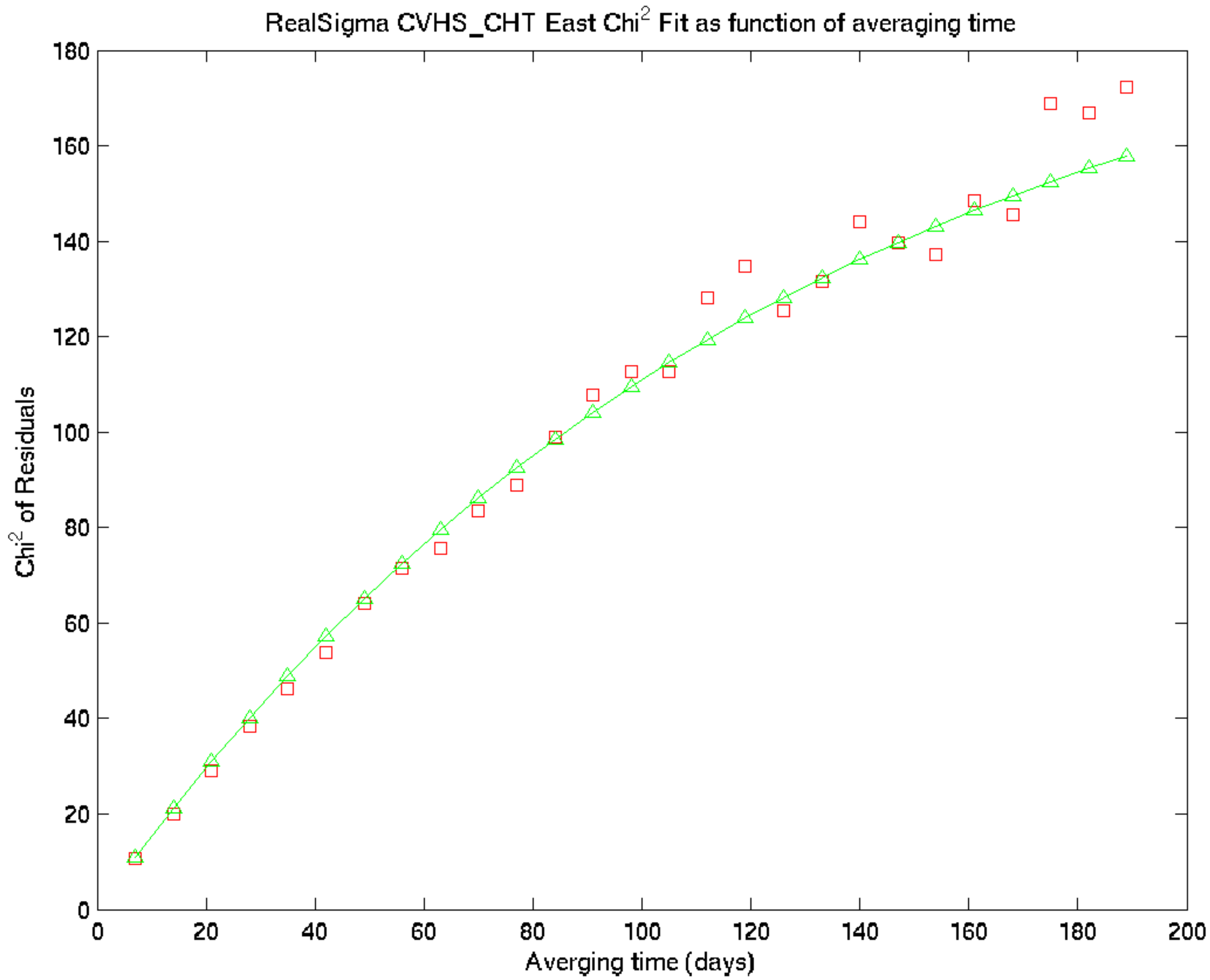


100-d avg
wrms 0.6 mm
nrms 3.4



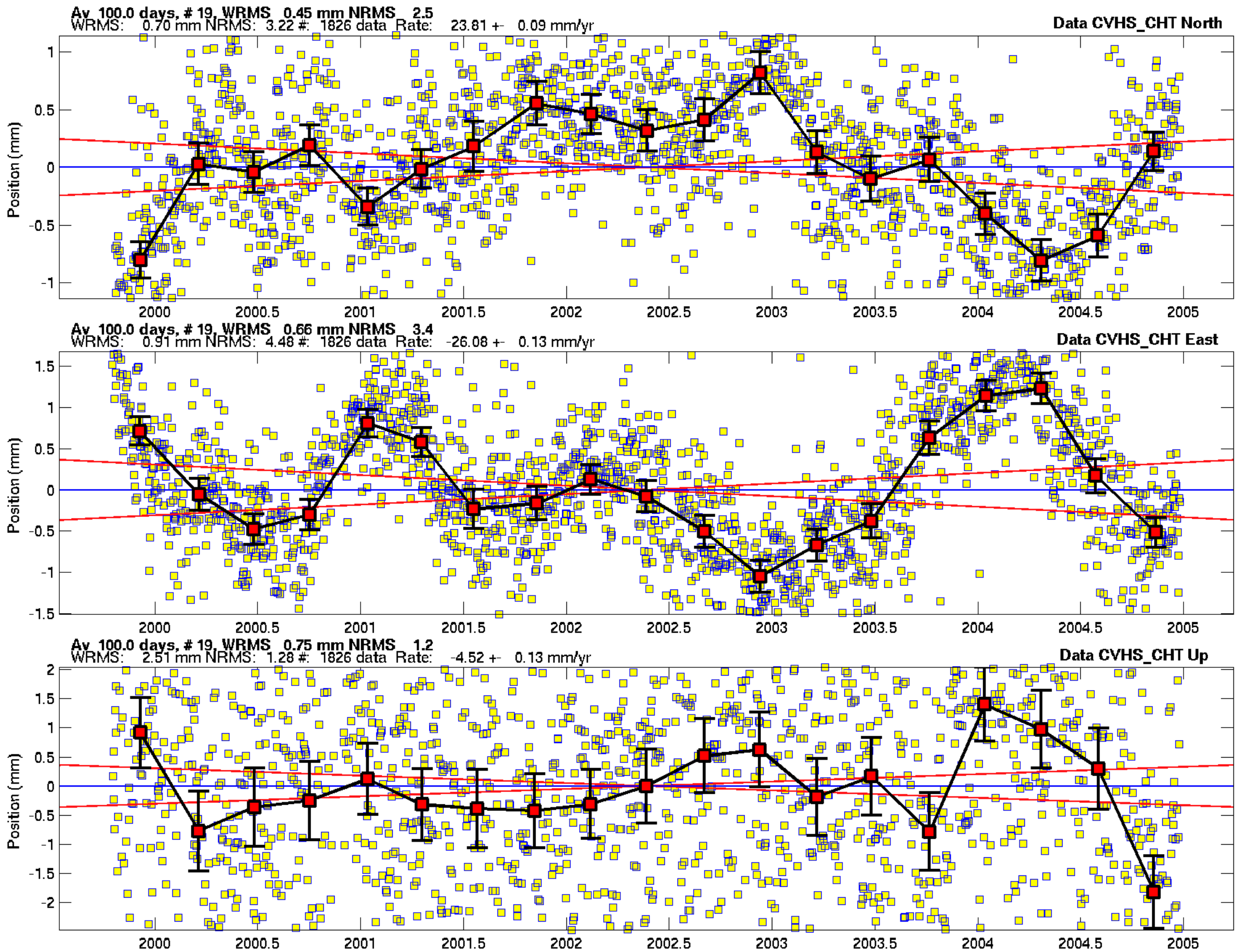
400-d avg
wrms 0.3 mm
nrms 3.1

Estimating a “realistic-sigma” by fitting an exponential function to chi-square vs averaging time



Get scale factor by evaluating the function at an infinite averaging time

Using TSVIEW to compute and display the "realistic-sigma" results

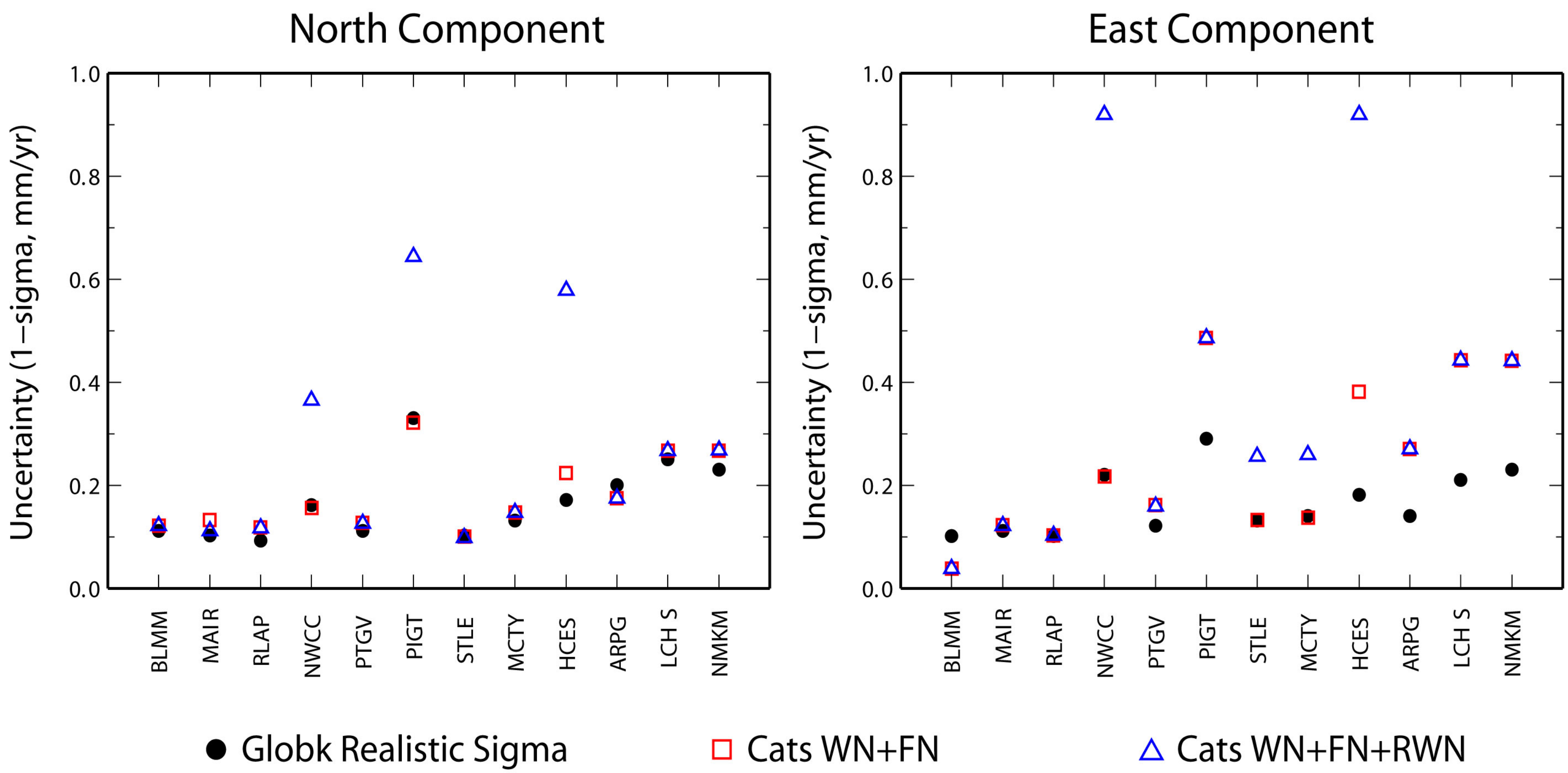


Note rate uncertainties with the "realistic-sigma" algorithm :

0.09 mm/yr N
0.13 mm/yr E
0.13 mm/yr U

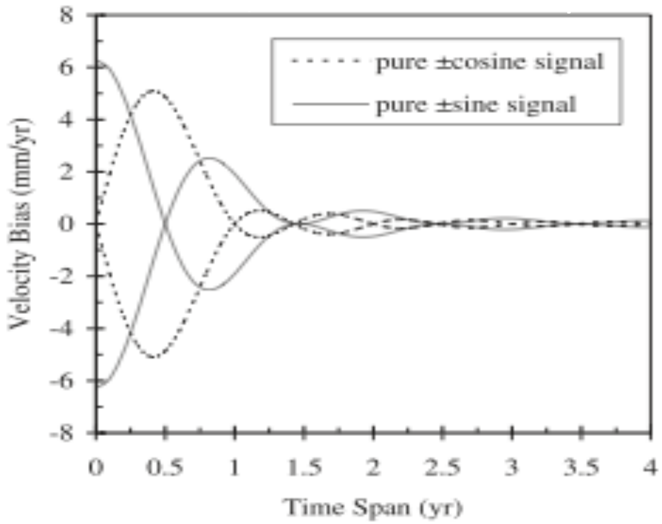
Red lines show the 68% probability bounds of the velocity based on the results of applying the algorithm.

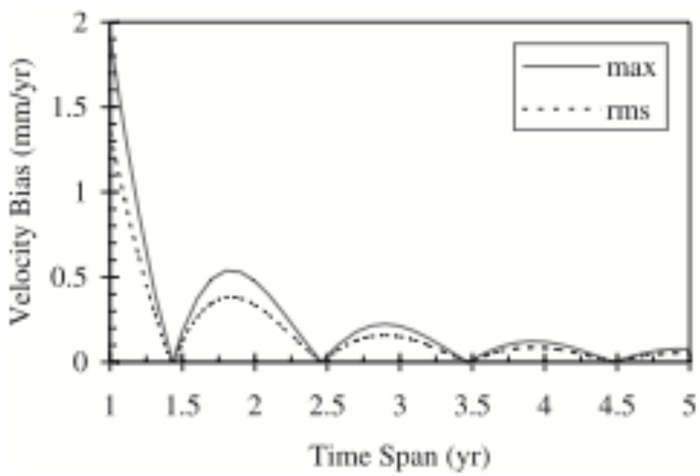
Comparison of estimated velocity uncertainties using spectral analysis (CATS*) and Gauss-Markov fitting of averages (GLOBK)



Plot courtesy E. Calias

Velocity Errors due to Seasonal Signals





Theoretical analysis of a continuous time series by *Blewitt and Lavallee [2002]*

Top: Bias in velocity from a 1mm sinusoidal signal in-phase and with a 90-degree lag with respect to the start of the data span

Bottom: Maximum and rms velocity bias over all phase angles

- The minimum bias is NOT obtained with continuous data spanning an even number of years
- The bias becomes small after 3.5 years of observation

Summary of Practical Approaches

- White noise + flicker noise (+ random walk) to model the spectrum [Williams et al., 2004]
- White noise as a proxy for flicker noise [Mao et al., 1999]
- Random walk to model to model an exponential spectrum [Herring "realistic sigma" algorithm for velocities]
- "Eyeball" white noise + random walk for non-continuous data

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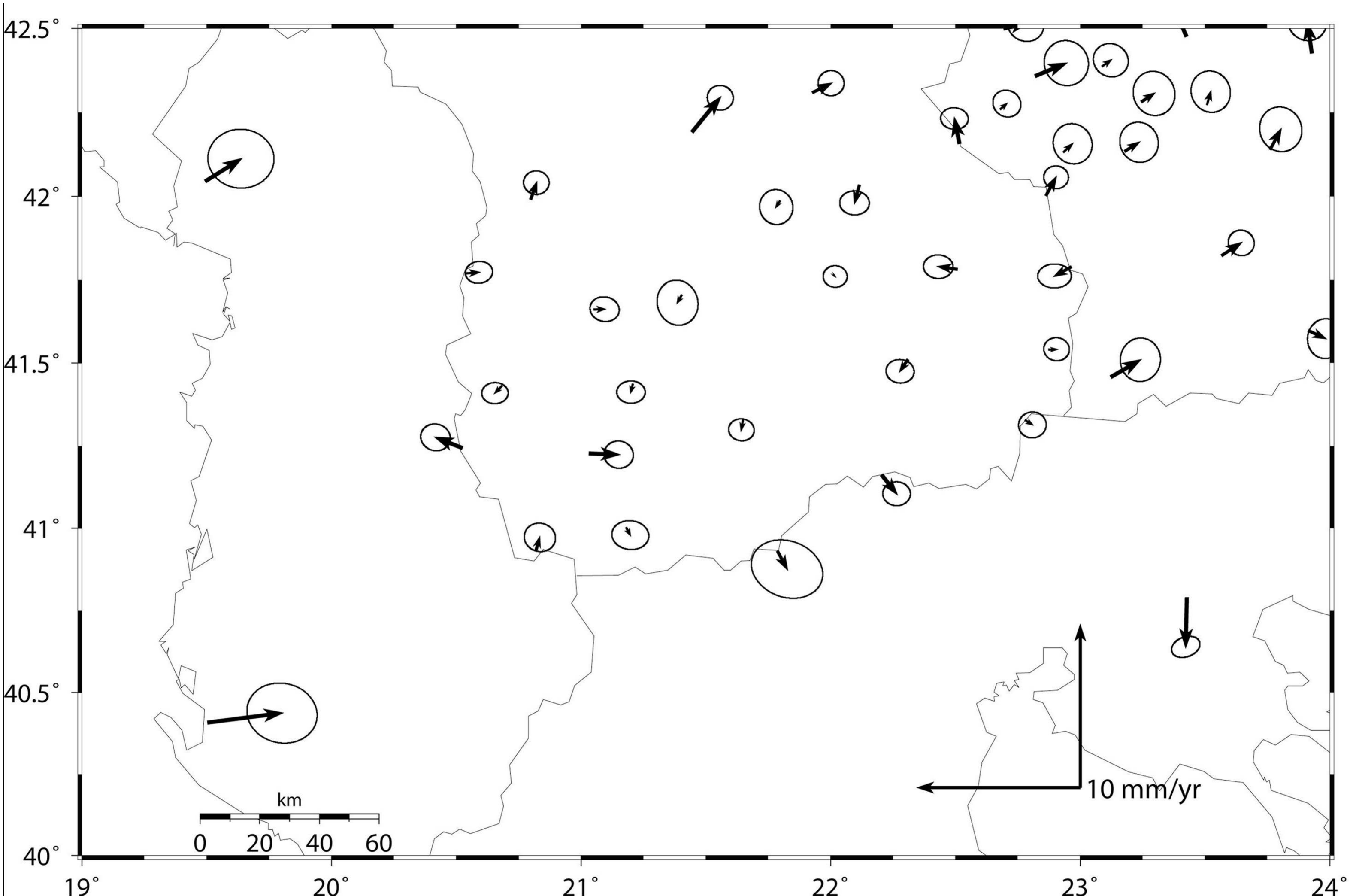
- Only the last two can be applied in GLOBK for velocity estimation
- All approaches require common sense and verification

Determining the Uncertainties of GPS Estimates of Station Velocities

- Understanding the sources of error
- Time series analysis to determine statistics for reweighting the data
- Whatever the assumed error model and tools used to implement it, external validation is important

External Validation for Velocity Uncertainties

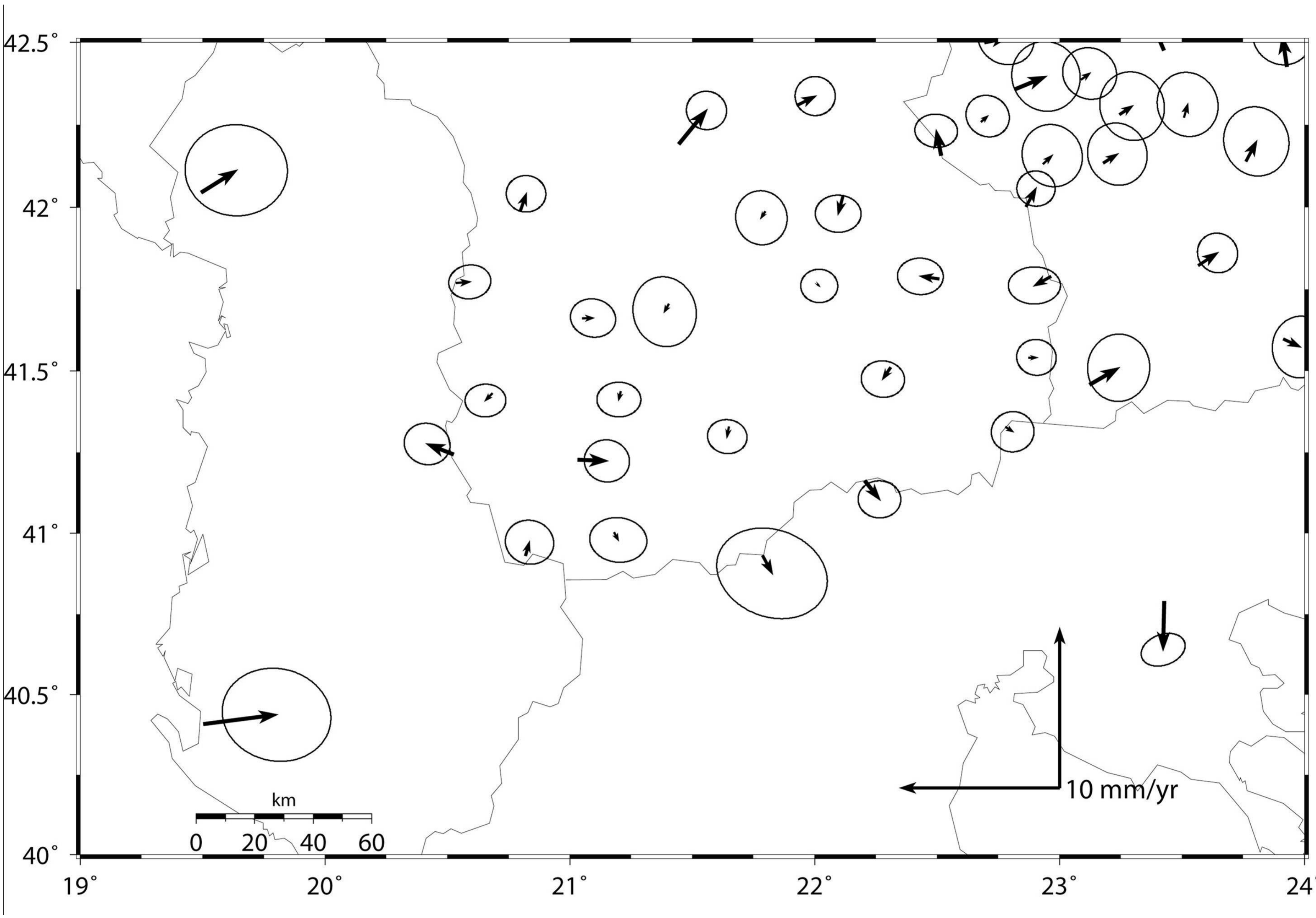
-- assume no strain within a geological rigid block



GMT plot at 70% confidence

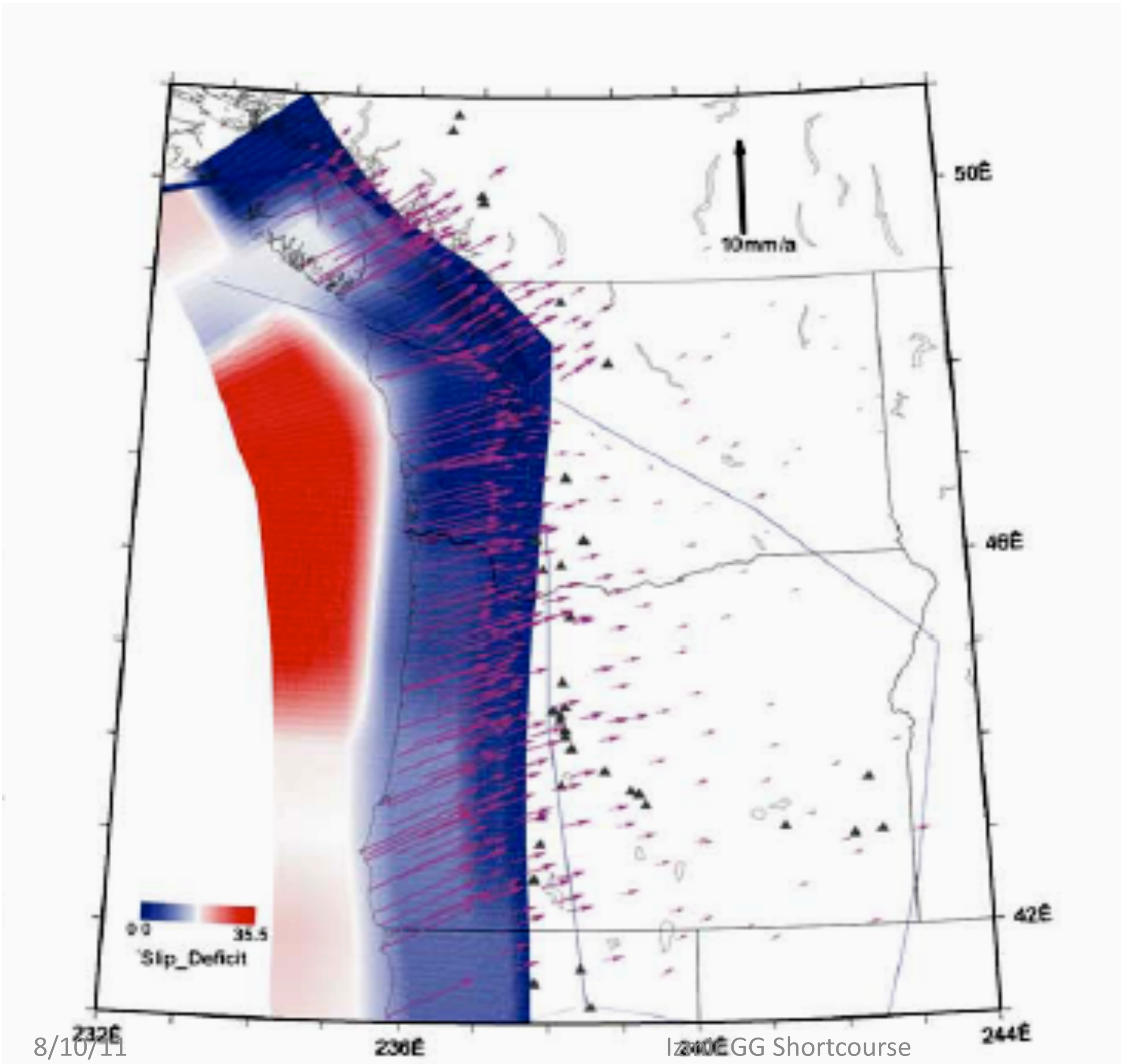
17 sites in central Macedonia: 4-5 velocities pierce error ellipses

.. same solution plotted with 95% confidence ellipses

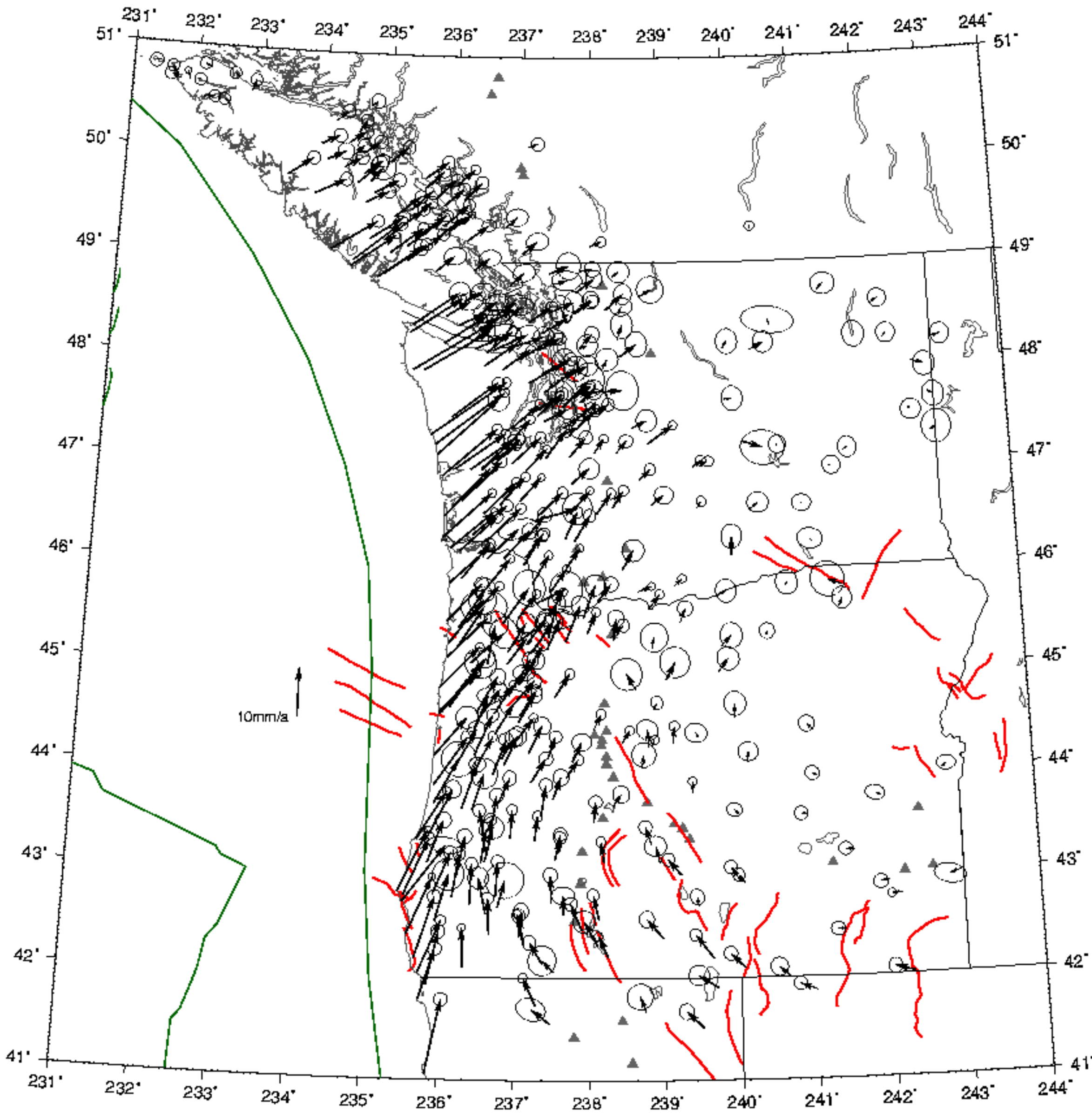


1-2 of 17 velocities pierce error ellipses

A more rigorous assessment for data from Cascadia

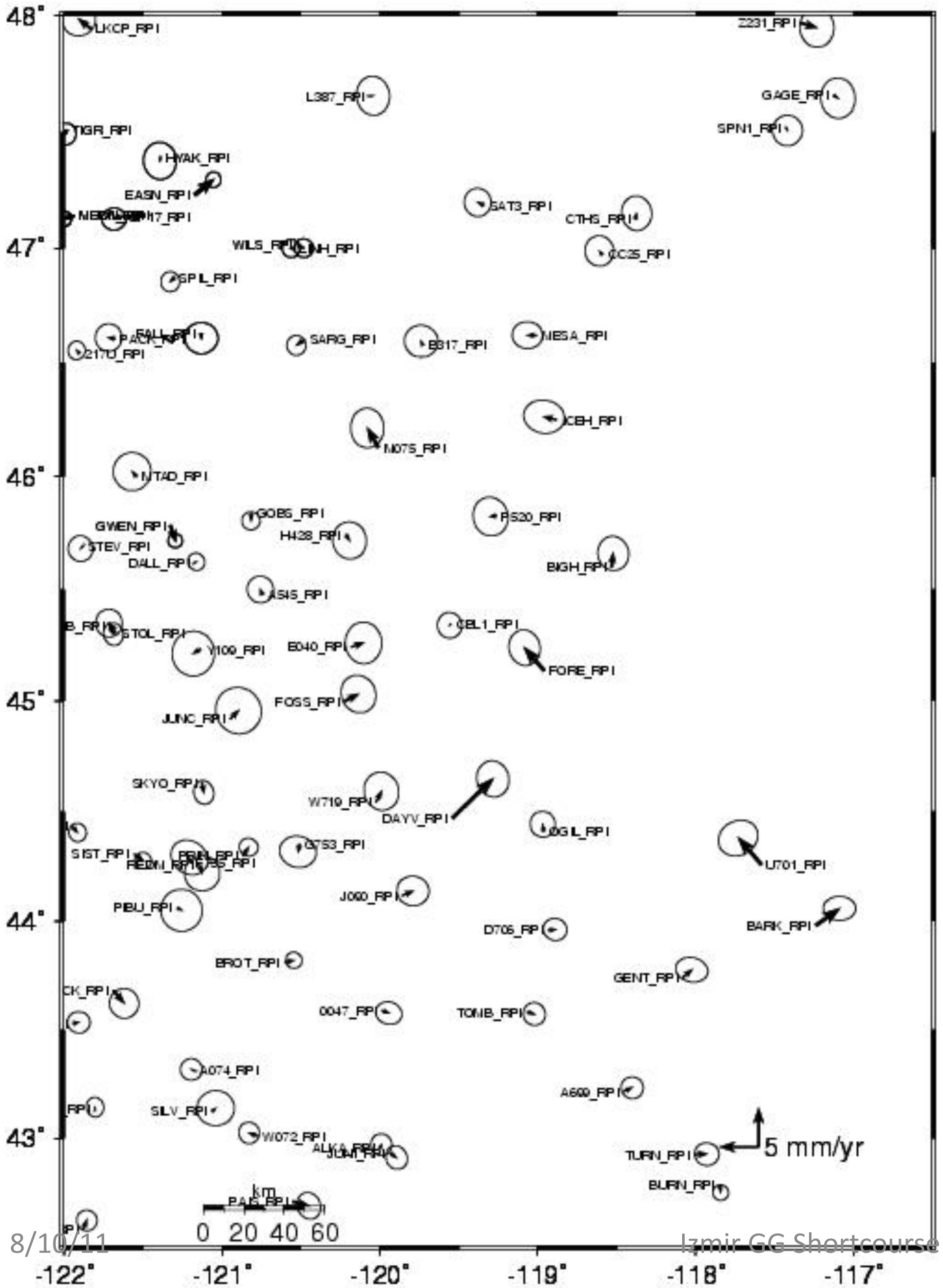


Colors show slipping and locked portions of the subducting slab where the surface velocities are highly sensitive to the model; area to the east is slowly deforming and insensitive to the details of the model



Velocities and 70% error ellipses for 300 sites observed by continuous and survey-mode GPS 1991-2004

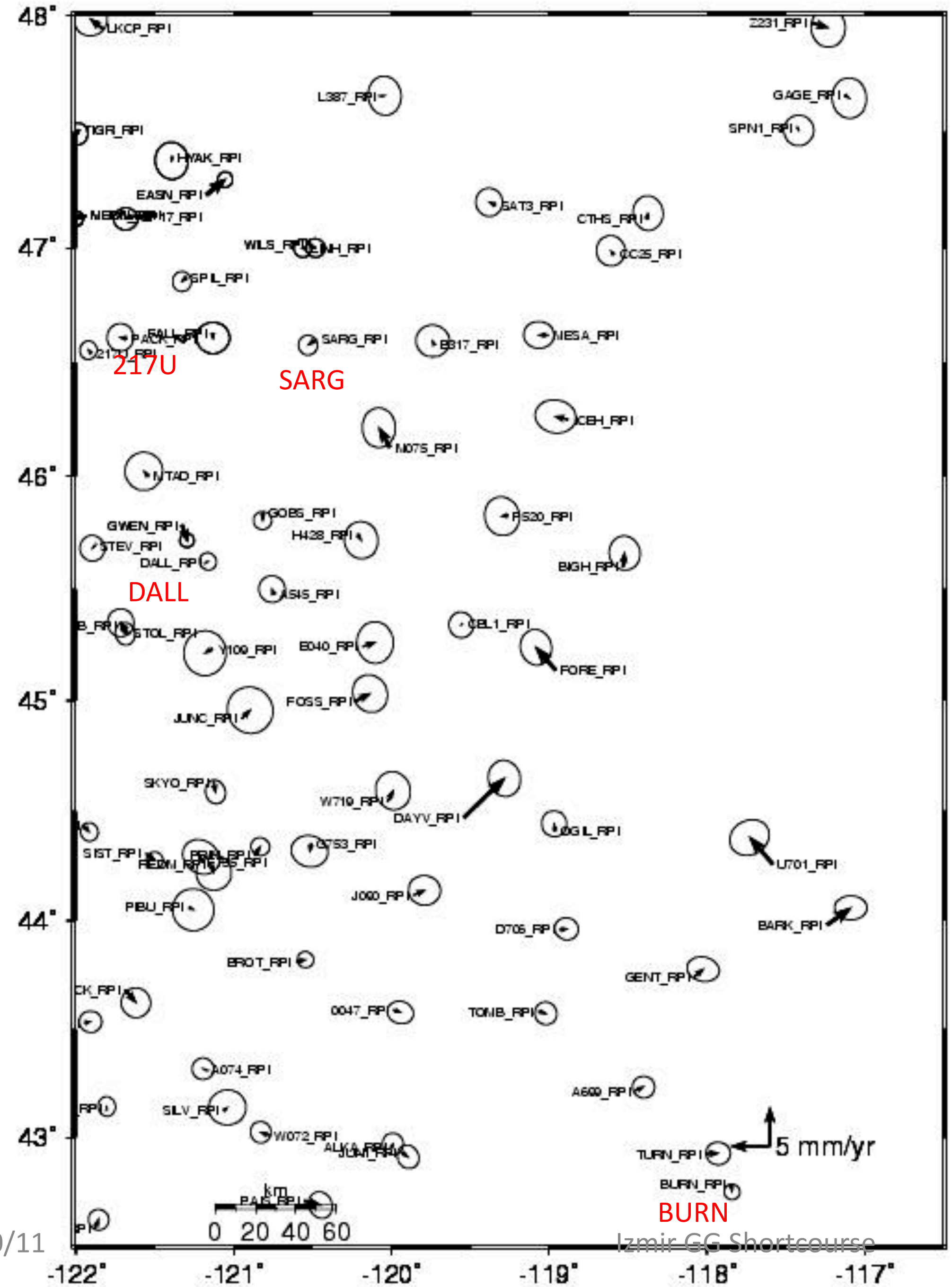
Test area (next slide) is east of 238E



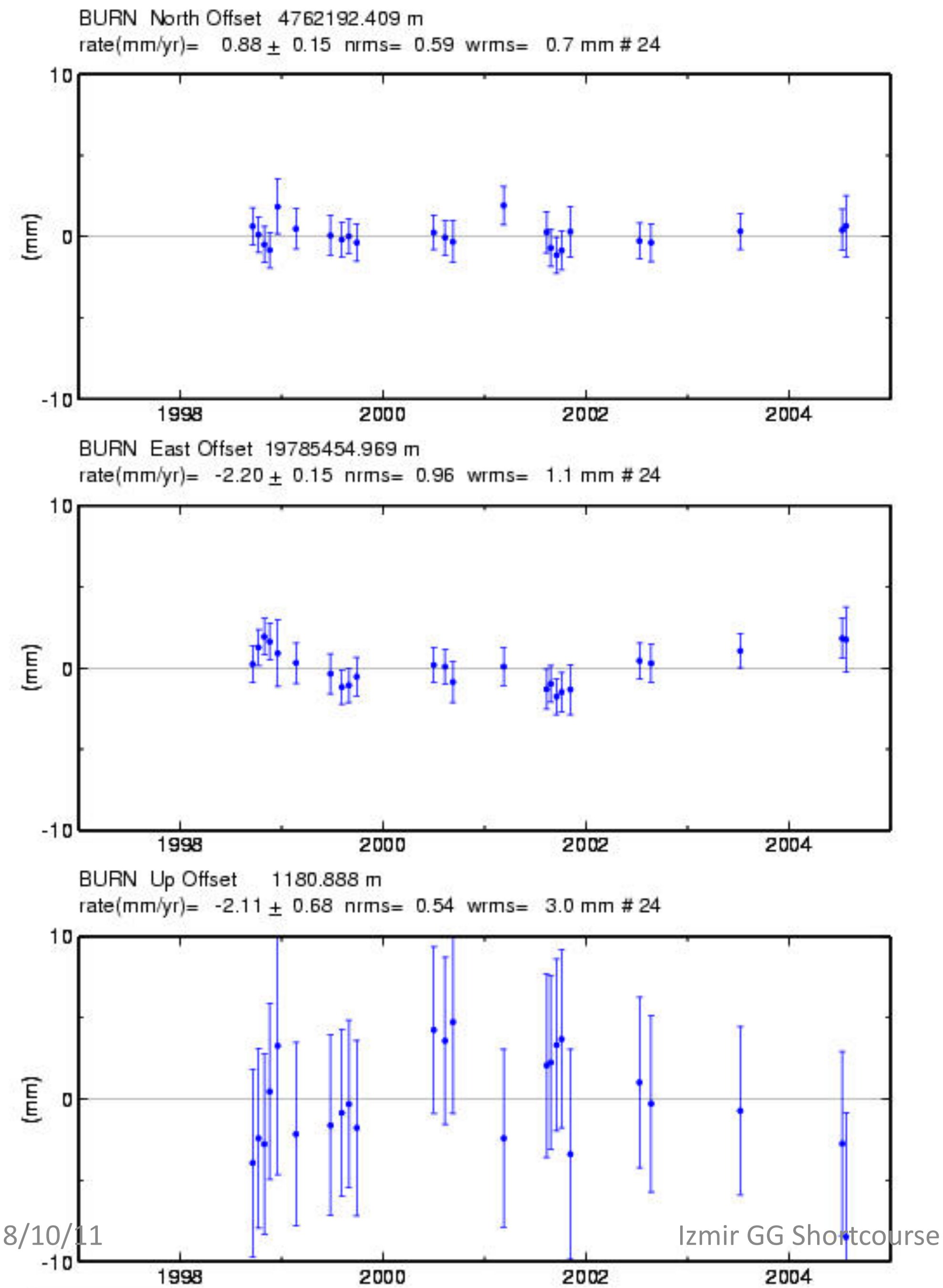
Residuals to elastic block model for 70 sites in slowly deforming region

Error ellipses are for 70% confidence:

13-17 velocities pierce their ellipse



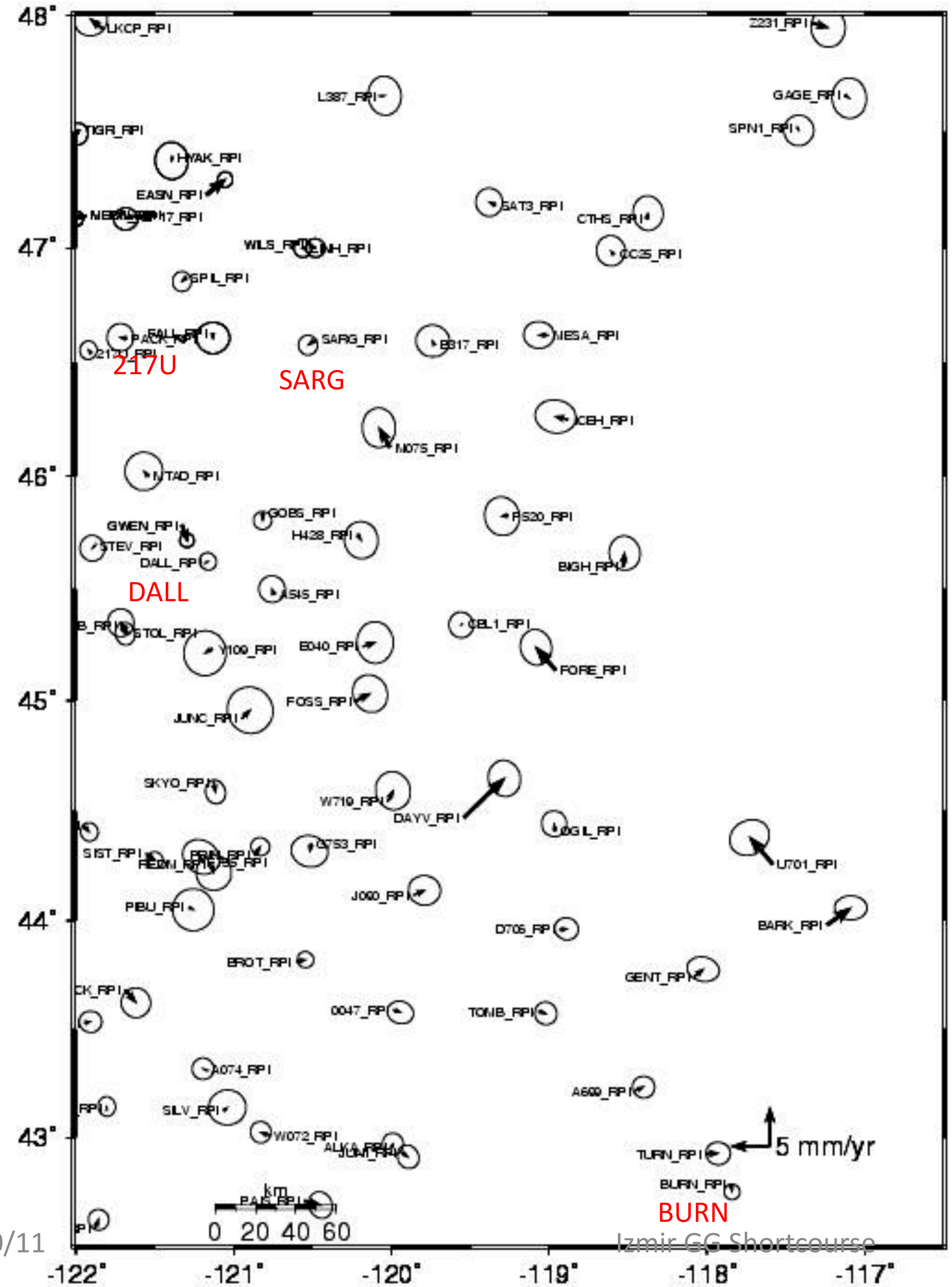
Locations of one continuous (BURN) and 3 survey-mode sites for time series shown in next slides



Time series of monthly position estimates for continuous site BURN

Wrms ~ 1 mm N,E
~ 3 mm U

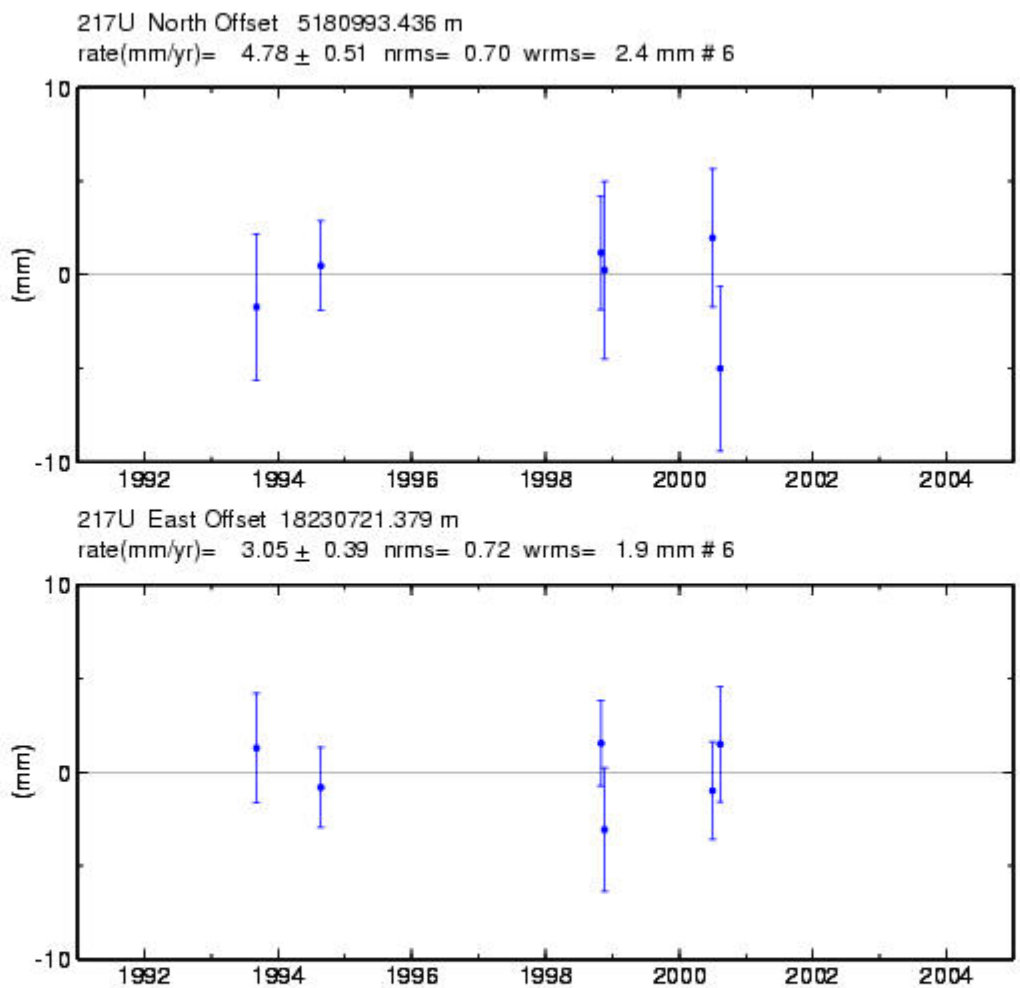
Rate uncertainties
< 0.2 mm/yr N,E
0.7 mm/yr U
do not include random walk added for velocity estimates



Next slide shows time series for survey-mode site 217U

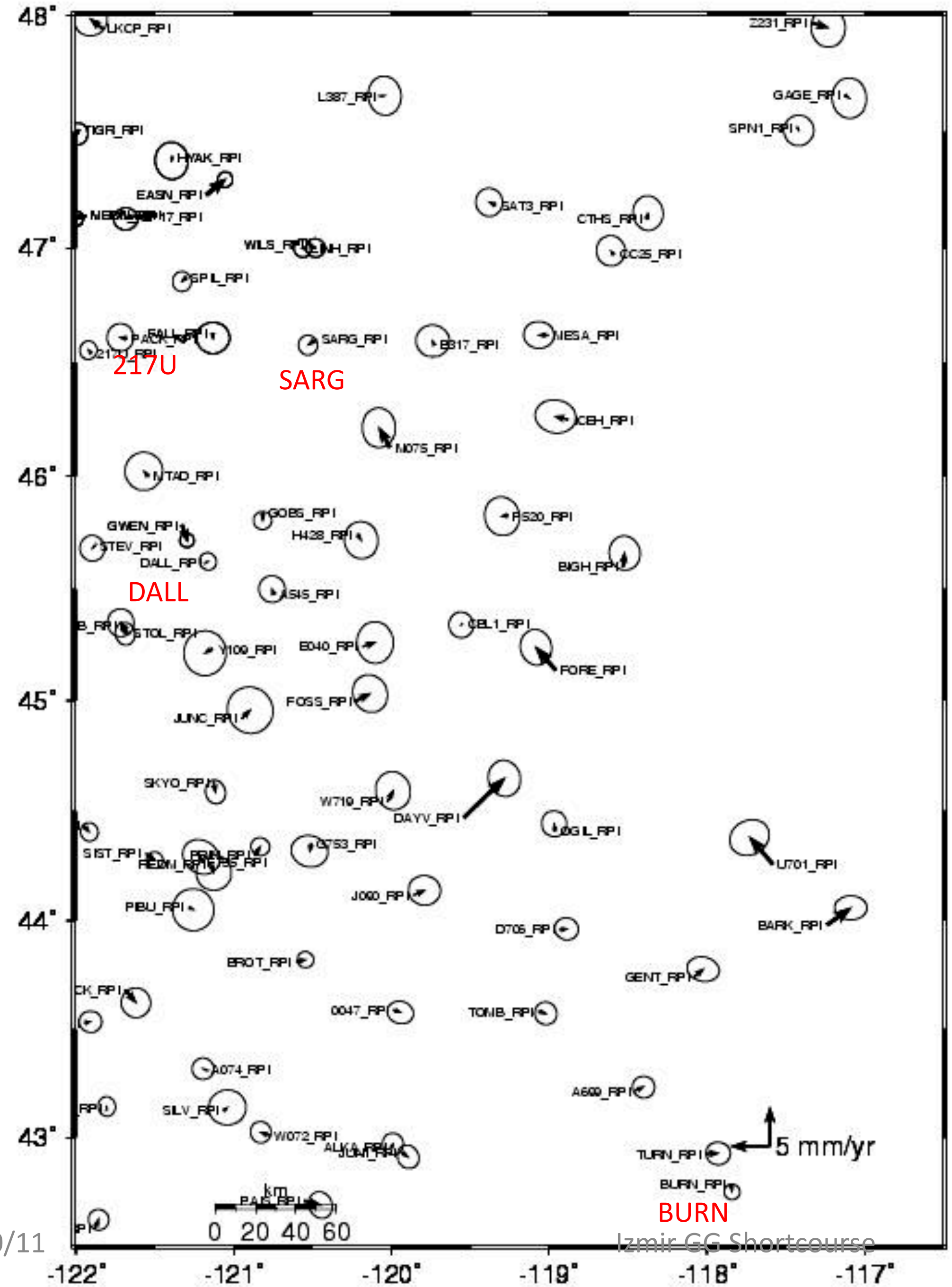
Note consistency with nearby sites

Time series for survey-mode site 217U



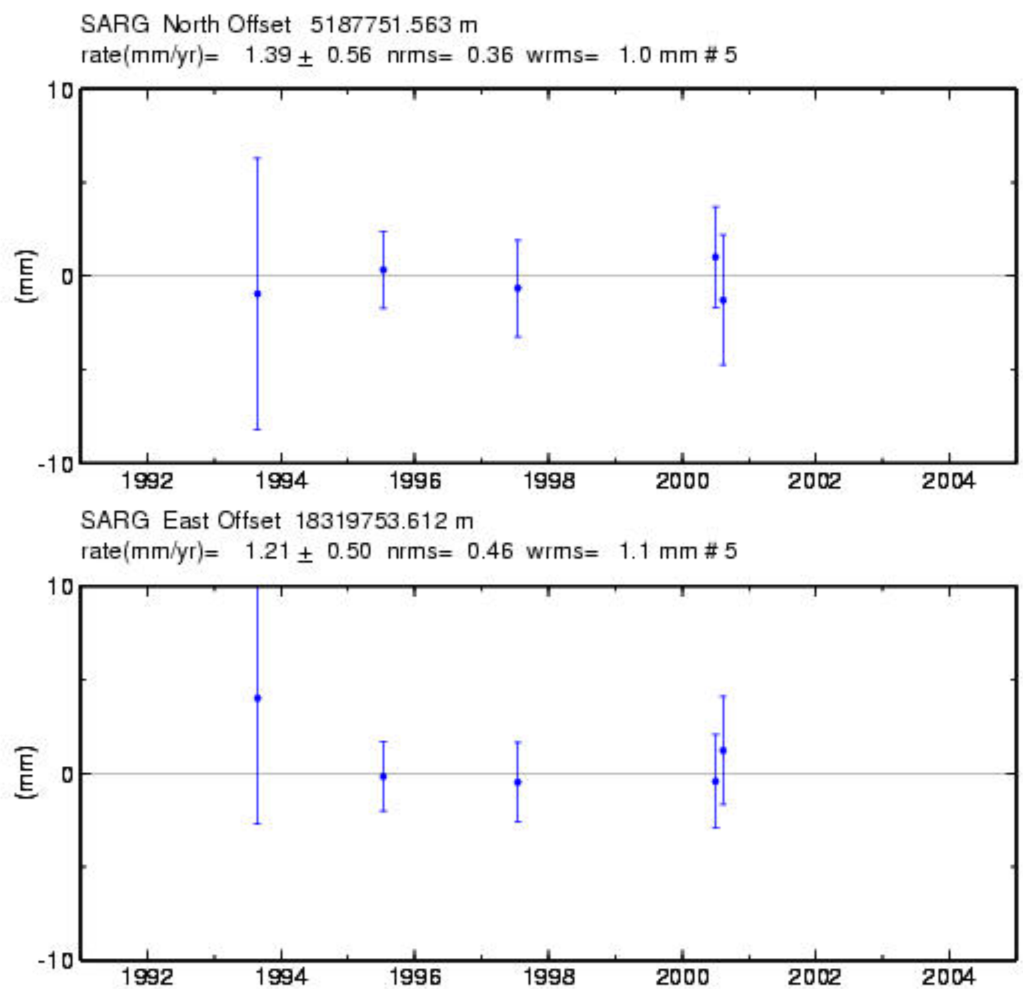
Position estimates based on 8-24 hr occupations

Note < 1 mm rate uncertainties due to 7-yr time span



Next slide shows time series for survey-mode site SARG

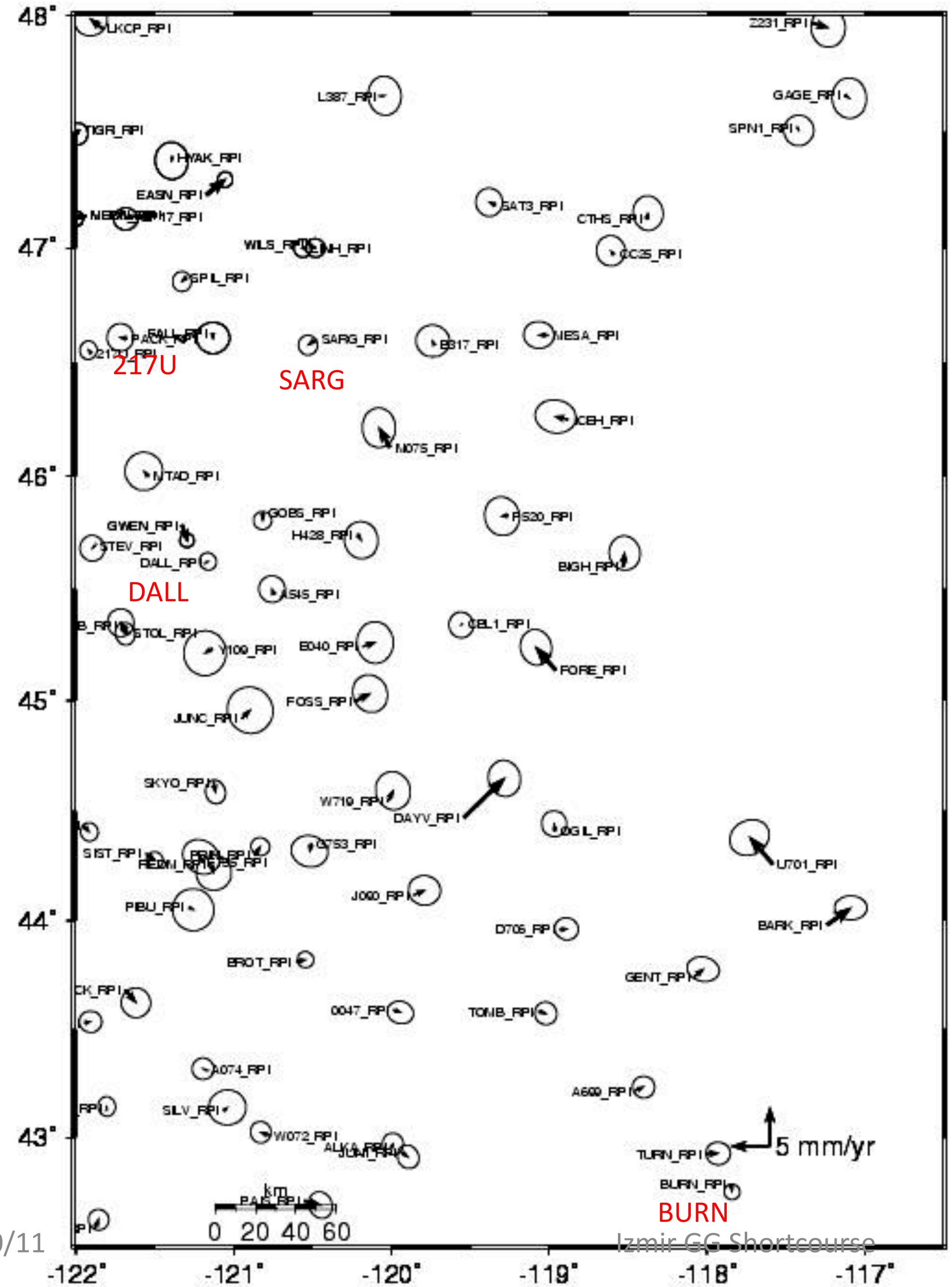
Horizontal time series for survey-mode site SARG



Position estimates based on 8-24 hr occupations

Note 1 mm wrms and

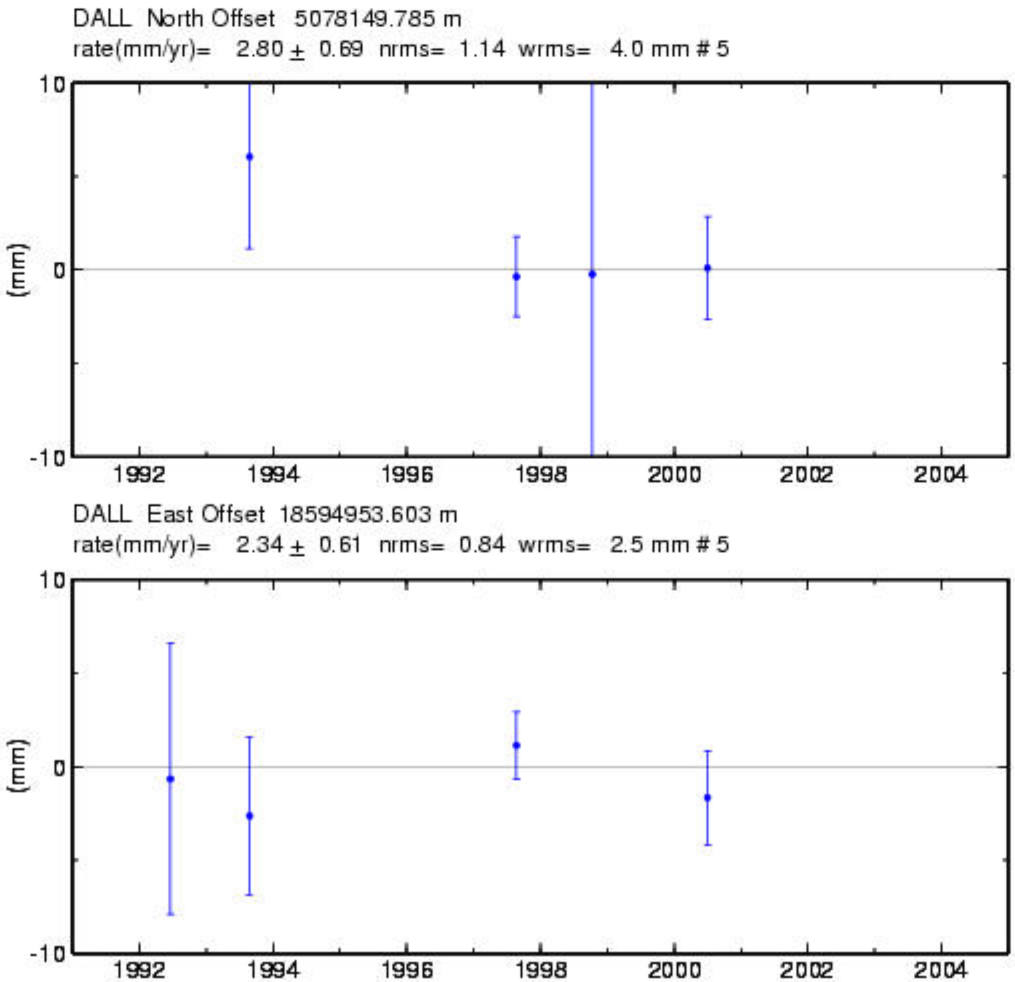
< 1 mm rate sigmas



Next slide shows time series for survey-mode site DALL

Note consistency with nearby sites except continuous site GWEN

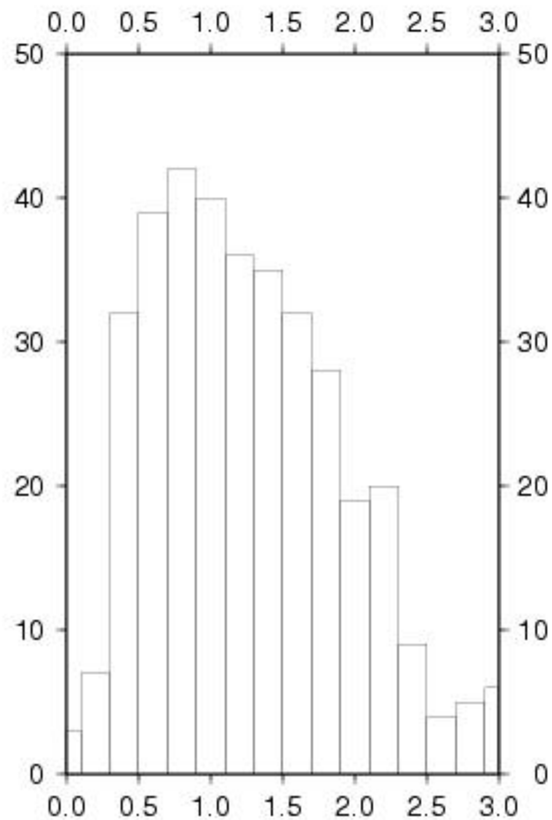
Horizontal time series for survey-mode site DALL



Position estimates based on 8-24 hr occupations

Rate sigmas < 1 mm/yr and consistent with surrounding sites even with velocities determined essentially by two occupations 3 yrs apart

Statistics of Time Series



NRMS

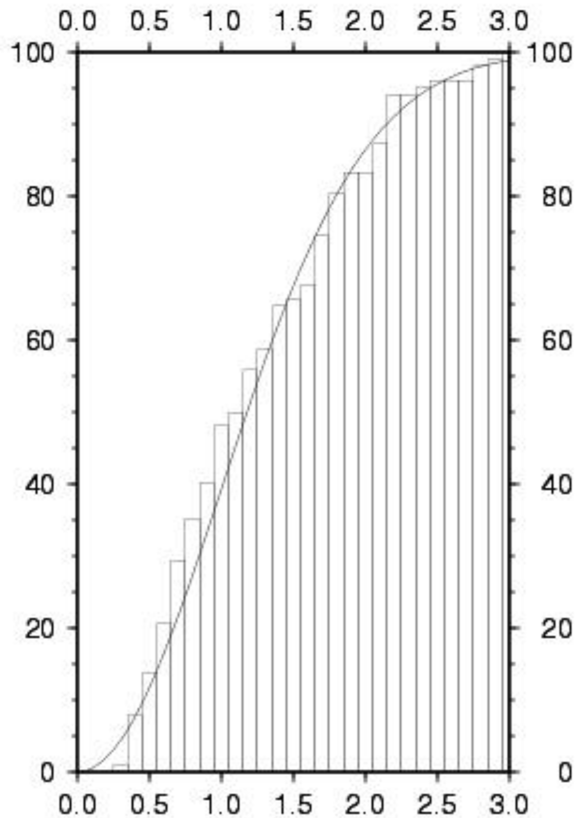
Distribution of normalized rms for horizontal magnitudes residuals after removing the block model

357 sites

NRMS
E, N 1.00, 1.03

Statistics of Velocity Residuals

Percent
Within
Ratio



Ratio (velocity magnitude/uncertainty)

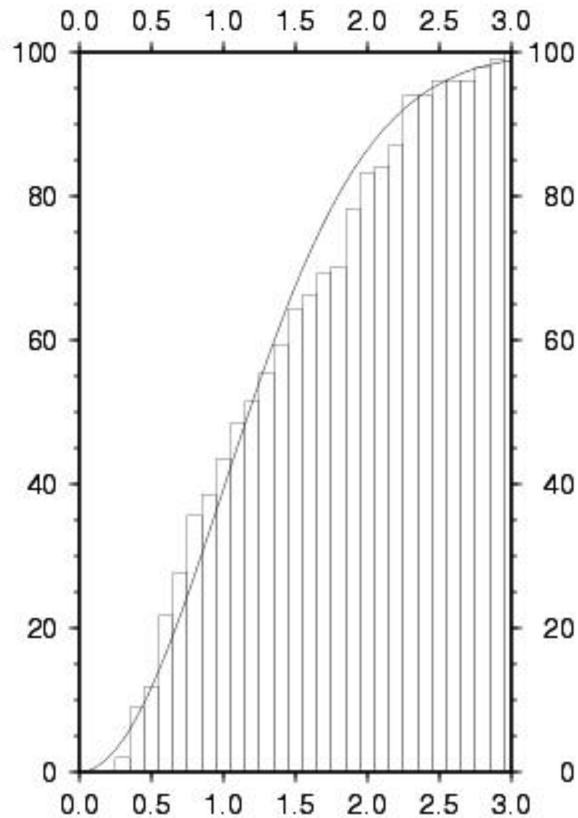
Cumulative histogram of normalized velocity residuals for Eastern Oregon & Washington (70 sites)

Noise added to position for each survey:
0.5 mm random
1.0 mm/sqrt(yr)) random walk

Solid line is theoretical for Gaussian distribution

Statistics of Velocity Residuals

Percent Within Ratio



Ratio (velocity magnitude/uncertainty)

Same as last slide but with a smaller random-walk noise added :

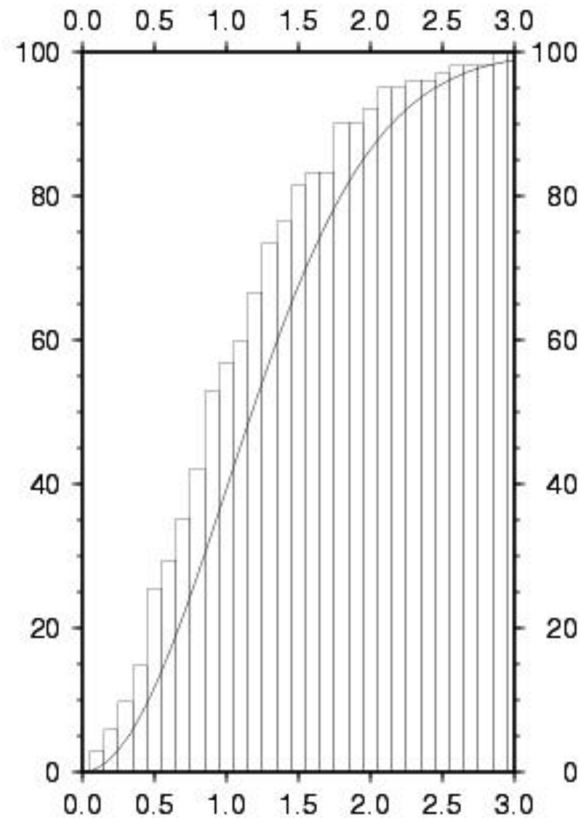
0.5 mm random
0.5 mm/yr random walk

(vs 1.0 mm/sqrt(yr)) RW for ‘best’ noise model)

Note greater number of residuals in range of 1.5-2.0 sigma

Statistics of Velocity Residuals

Percent
Within
Ratio



Ratio (velocity magnitude/uncertainty)

Same as last slide but with larger random and random-walk noise added :

2.0 mm white noise
1.5 mm/sqrt(yr)) random walk

(vs 0.5 mm WN and 1.0 mm/sqrt(yr)) RW for 'best' noise model)

Note smaller number of residuals in all ranges above 0.1-sigma

Summary

- All algorithms for computing estimates of standard deviations have various problems: Fundamentally, rate standard deviations are dependent on low frequency part of noise spectrum which is poorly determined.
- Assumptions of stationarity are often not valid
- "Realistic sigma" algorithm is a covenient and reliable appraoch to getting velocity uncertainties in globk
- Velocity residuals from a model, together with their uncertainties, can be used to validate the error model

Tools for Error Analysis in GAMIT/GLOBK

- GAMIT: AUTCLN reweight = Y (default) uses phase rms from postfit edit to reweight data with constant + elevation-dependent terms
- GLOBK
  - rename (eq_file) _XPS or _XCL to remove outliers
  - sig_neu adds white noise by station and span; useful for handling outliers
  - mar_neu adds random-walk noise: principal method for controlling velocity uncertainties
  - In the gdl files, can rescale variances of an entire h-file: useful when combining solutions from with different sampling rates or from different programs (Bernese, GIPSY)
- Utilities
  - Realistic sigma" algorithm implemented in tsview (MATLAB) and enfit/ensum; sh_gen_stats generates mar_neu commands for globk based on the noise estimates
  - sh_plotvel (GMT) allows setting of confidence level of error ellipses
  - sh_tshist and sh_velhist can be used to generate histograms of time series and velocities

Summary

- There are no absolute methods that ensure the correct error model can be determined for a set of data processing.
- We attempt to determine with 1-sigma values, that 68% of values will be within this range due to noise; with 2-sigma, 95% of values (1-d) even when the probability distribution is not Gaussian.
- The most under certain aspect is determining the nature of the temporal and spatial correlations in the results. Generally, large amounts of data are needed for this and the assumption of stationarity.

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